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Journal of Research
of the
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MATHEMATICAL TABLES

A list of the mathematical tables now obtainable from the National Bureau of Standards is printed on pages 3 and 4 of the JOURNAL cover. New tables will be announced from time to time as they become available.

U. S. Department of Commerce National Bureau of Standards

RESEARCH PAPER RP1549

Part of Journal of Research of the National Bureau of Standards, Volume 31, August 1943

TRISTIMULUS SPECIFICATION OF THE MUNSELL BOOK OF COLOR FROM SPECTROPHOTOMETRIC MEASUREMENTS

By Kenneth L. Kelly,[1] Kasson S. Gibson, and Dorothy Nickerson [2]

ABSTRACT

The development of the Inter-Society Color Council-National Bureau of Standards (ISCC-NBS) system of color names, based on the standards in the Munsell Book of Color, made it necessary to specify the master standards of this book in fundamental terms. Accordingly, spectral reflection curves were run for each of the 421 master standards on the General Electric recording spectrophotometer at the National Bureau of Standards, using slit widths of approximately 4 millimicrons. Various corrections were applied to these spectrophotometric data in accordance with methods regularly used for such work at the bureau. Colorimetric computations were then made with these data, resulting in tristimulus specifications according to the 1931 ICI standard observer and coordinate system. Four illuminants were used: ICI illuminants *A* and *C*, representative of incandescent-lamp light and average daylight, respectively, illuminant "*D*" (lightly overcast north sky), and illuminant "*S*" (extremely blue sky). The colorimetric specifications of the Munsell standards for all four illuminants are thus given.

The trilinear coordinates for the Munsell standards calculated for ICI illuminant *C* have been plotted on large chromaticity (x, y) diagrams and constant Munsell chroma lines drawn in. (Similar values obtained by Glenn and Killian at the Massachusetts Institute of Technology in 1935 for Munsell color standards bearing the same hue-value-chroma designations have also been plotted on the diagram and differences between the two sets of data are discussed.) These diagrams serve as means for determining the Munsell notation and thereby the ISCC-NBS color name for any color whose trilinear coordinates and apparent reflectance are given.

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I. INTRODUCTION

Two of the official compendia of drugs and medicines, the United States Pharmacopoeia and the National Formulary, specify the purity and quality of drugs by a number of tests for which tolerance limits are set. With a crude drug, for example, these tests refer to ash, acid

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insoluble ash, size, chemical identification tests, taste, color, and so forth, these being indications of purity or quality. All of the tests except color have been under continuous study by committees entrusted with their revision. Color, on the other hand, presented a different type of problem whose solution was not attempted until 1931. Previously the color terms used in the USP and NF had enjoyed no official definition but contained, among others, such confusing terms as "brownish green" or "blackish white," with seldom any reference to a color chart or standard. In the monograph of a drug, the pharmacognocist describes the colors of the outside and the inside, the colors of the various microscopic elements, and finally, the colors of the identification tests. In each instance, no mention is made of the normality of the observer's color vision [1], [3] or of the conditions of lighting or viewing.

Agitation toward research for the development of a suitable system of color terminology was begun in the twenties by E. N. Gathercoal, then a member of the USP Revision Committee [2]. After the founding of the Inter-Society Color Council, of which he was the first chairman, studies were made of the then existing color systems, and in 1933 the report [3] was submitted which became the basis of the system of color names now known as the ISCC-NBS system of color names [4]. Procedures were developed at the same time for the application of these color names to the description of the colors of crude drugs, powdered drugs, chemicals, liquids, precipitates, microscopic structures, and fluorescent materials [5]. The central notations of the color-name blocks were determined for the application of these color names to the description of the colors of soils [6]. Recently these names have also been used to describe the colors of illuminants, and a description of this method of use is in preparation.

In all of this work, the boundaries of the separate color-name blocks have been specified in terms of the Munsell color standards [7, 8]. It was realized early in the project that in order to be placed on a sound basis the individual boundaries must be specified in fundamental terms. The accuracy of the system of color names would then be independent of the existence or stability of the individual system of material color standards, in terms of which the system is used in practice. Since the Munsell color system provided a very satisfactory means of determining which color name best described the color of an object, it was decided to measure the spectral reflectances of all of the color standards in the Munsell Book of Color. The specification of the trilinear coordinates and apparent reflectances of each of the Munsell samples would provide an invariable specification of the color of that sample and thereby of a definite point in the framework of the system by which the relative position of each color name is indicated.

Tristimulus specifications of the Munsell Book of Color have been published by Glenn and Killian [8] and were available for some time before that date. Instead of using the Glenn-Killian data, however, it seemed preferable to define the ISCC-NBS system of color names by way of the Munsell samples actually used in the color-names work. This involved a nominal repetition of the spectrophotometric and colorimetric work carried out by Glenn and Killian, but avoided uncertainties arising out of the possible differences between the respective Munsell samples bearing the same

[3] Figures in brackets indicate the literature references at the end of this paper.

color designation as well as those arising from the unknown history and usage of the Glenn-Killian samples prior to their measurement. Furthermore, the present authors desired to use in the spectrophotometric measurements certain methods of calibration regularly used at the National Bureau of Standards for such work. The measurements and computations described below were accordingly undertaken, and the diagrams and tables included in the present paper provide a means by which a color may be named without reference to a color chart, or by which the boundaries of the color-name blocks may be specified in terms of a fundamental color system. It is now possible to select the appropriate color name for a color when the fundamental specifications for that color are given.

Since the application of this system of color names will be made in the plant or in the field where the illumination used will usually be daylight, all of the techniques and computations, both for the color names and for the Munsell system, have primarily been made on the basis of ICI illuminant *C*. However, colorimetric data on the Munsell standards for other illuminants are also of interest. Accordingly, based on the same spectrophotometric data, tristimulus values have been computed for four illuminants—ICI illuminant *C* [9] (representative of average daylight), ICI illuminant *A* [9] (2,842°K [10], representative of incandescent illuminants), illuminant *"D"* [11, 12] (representative of lightly overcast north sky), and illuminant *"S"* [13, 14] (representative of extremely blue sky).

II. SAMPLES MEASURED

Prior to his death, Walter T. Spry, then manager of the Munsell Color Co., deposited one or more samples of all of the original paintings of the standards in the Munsell Book of Color with the Colorimetry Section of the Bureau. He also deposited repaints of all colors the original paintings of which had become depleted, together with new colors prepared up to 1935. In selecting the samples of each color to be measured, that painting was chosen which matched the color chip of the same designation in the Munsell Book of Color. In most instances the color differences between the originals and their repaints were negligible, but in several cases it was important to specify which painting was used. Therefore, for the purpose of accuracy and as a matter of record, the painting number of each sample measured is given.

The 2-value 2-chroma samples for the intermediate hues (*10R, 10YR, 10Y*, etc.) were painted independently of the other 2-value 2-chroma samples, and the colors and the data are not as congruent with the other samples as they are with each other. These samples, as well as several 8-value 2-chroma samples for the intermediate hues, are not included in the Munsell Book of Color, but they were measured and the data are included in the present paper for the sake of completeness. One new sample, *10YR 8/8*, recently received, is included. The complete list of samples measured is given in table 2.

The samples in the Munsell Book of Color were inspected under a strong source of ultraviolet radiant energy and also under a strong yellowish green light for fluorescence that might vitiate the spectrophotometric measurements [15]. No fluorescence was observed under either illuminant.

III. METHODS OF MEASUREMENT AND COMPUTATION

Spectral reflection curves of all of the samples noted and listed in table 2 were run on the General Electric recording spectrophotometer at the National Bureau of Standards. The samples were run relative to magnesium oxide [16], with approximately 4 mμ slits and over a wavelength range from 400 to 750 mμ. The samples were backed with black paper for these measurements. Calibration curves were run on each sheet, enabling corrections to be applied to the data for wavelength errors, for 100-percent and zero curve deviations, and for aging of the magnesium oxide comparison surface, in accordance with methods regularly used at the National Bureau of Standards [17, 18].

As already noted, the colorimetric computations were made for four different illuminants. ICI illuminants *A* and *C* have become well established in colorimetric work. Illuminant *A* is the Plankian radiator or blackbody at 2,842° K (C_2=14,320 micron-degrees, or 2,848° K with C_2=14,350); the color temperatures of common incandescent illuminants vary from about 2,600° to about 3,100° K. Illuminant *C* is that produced by a source at 2,842° K combined with a certain Davis-Gibson daylight liquid filter [19]. On the "OSA excitations" basis (used in the design of the Davis-Gibson filters) the resulting color matched that of a Plankian radiator at 6,500° K. On the basis of the ICI data the approximate color temperature of this lamp-and-filter combination is 6,800° K. The color and spectral-energy distribution of ICI illuminant *C* satisfactorily match those of overcast sky or average daylight for colorimetric use. Illuminant "*D*" is that produced by an illuminant at 3,000° K combined with a Macbeth (Corning) daylight glass filter giving a color temperature of approximately 7,500° K. The color of illuminant "*D*", found to be the optimum color for cotton grading, is also being widely used for agricultural grading and textile color matching. Its color closely matches that of the lightly overcast north sky most desired for such work. Illuminant "*S*" was designed as the blue end point for a series of illuminants representing the range from fully overcast to maximally clear sky. It was devised by weighting Abbot's "sun-outside-atmosphere" energy data by the inverse λ^4 scattering relation. Illuminant "*S*" has been designated as "limit blue sky."

The colorimetric data on the Munsell samples for ICI illuminant *C*, representative of average daylight, are of primary interest and the computations were carried out both at the National Bureau of Standards and in the U. S. Department of Agriculture. Those for the other three illuminants were made in the Department of Agriculture. All of the computations in the Department of Agriculture were done by using Hollerith cards and automatically punching sums obtained by the method of progressive digiting. The authors are indebted to Lila F. Knudsen, mathematical statistician of the Food and Drug Administration, for suggesting this rapid method of computation [20]. All of the computations were made by the weighted ordinate method.

The spectral-energy distributions of the four illuminants are shown in figure 1, and in table 1 are given the tristimulus data for the spectrum of each of the four illuminants used in the computations of *X*, *Y*, *Z* and *x*, *y*, *z*.

TABLE 1.—*ICI tristimulus data for the four illuminants, A, C, "D," and "S," used in deriving the colorimetric data on the Munsell standards*

Wavelength	For illuminant *A*			For illuminant *C*			For illuminant "*D*"			For illuminant "*S*"		
	$\bar{x}E$	$\bar{y}E$	$\bar{z}E$	$\bar{x}E$	$\bar{y}E$	$\bar{z}E$	$\bar{x}E$	$\bar{y}E$	$\bar{z}E$	$\bar{x}E$	$\bar{y}E$	$\bar{z}E$
Mμ												
380	1	------	6	4	------	20	6	------	30	36	------	165
390	5	------	23	19	------	89	27	1	128	99	3	473
400	19	1	93	85	2	404	119	3	567	349	10	1658
410	71	2	340	329	9	1570	446	12	2125	1199	33	5719
420	262	8	1256	1238	37	5949	1504	45	7223	3567	107	17137
430	649	27	3167	2997	122	14628	3373	138	16461	6852	280	33442
440	926	61	4647	3975	262	19938	4202	277	21077	8143	538	40845
450	1031	117	5435	3915	443	20638	4100	463	21613	7652	865	40332
460	1019	210	5851	3362	694	19299	3476	717	19952	6194	1278	35554
470	776	362	5116	2272	1058	14972	2274	1059	14982	3870	1803	25503
480	428	622	3636	1112	1618	9461	1070	1556	9099	1742	2533	14815
490	160	1039	2324	363	2358	5274	347	2258	5049	530	3444	7703
500	27	1792	1509	52	3401	2864	52	3451	2906	74	4871	4102
510	57	3080	969	89	4833	1520	96	5214	1640	127	6870	2160
520	425	4771	525	576	6462	712	626	7023	774	781	8757	965
530	1214	6322	309	1523	7934	388	1533	7986	391	1847	9618	471
540	2313	7600	162	2785	9149	195	2610	8574	182	2958	9717	207
550	3732	8568	75	4282	9832	86	4062	9324	82	4070	9343	81
560	5510	9222	36	5880	9841	39	6072	10162	40	5148	8615	34
570	7571	9457	21	7322	9147	20	8160	10194	22	6092	7610	16
580	9719	9228	18	8417	7992	16	9310	8840	17	6798	6454	13
590	11579	8540	12	8984	6627	10	8946	6599	10	7090	5229	7
600	12704	7547	10	8949	5316	7	8343	4956	6	6798	4038	5
610	12669	6356	4	8325	4176	2	7800	3913	2	5871	2945	2
620	11373	5071	3	7070	3153	2	6372	2841	1	4585	2044	1
630	8980	3704	------	5309	2190	------	4477	1847	------	3160	1303	------
640	6558	2562	------	3693	1443	------	2732	1067	------	2030	793	------
650	4336	1637	------	2349	886	------	1640	619	------	1183	447	------
660	2628	972	------	1361	504	------	988	365	------	636	236	------
670	1448	530	------	708	259	------	603	221	------	313	114	------
680	804	292	------	369	134	------	367	133	------	155	56	------
690	404	146	------	171	62	------	197	71	------	69	25	------
700	209	75	------	82	29	------	102	37	------	32	11	------
710	110	40	------	39	14	------	49	18	------	15	5	------
720	57	19	------	19	6	------	23	8	------	7	3	------
730	28	10	------	8	3	------	11	4	------	3	1	------
740	14	6	------	4	2	------	5	2	------	2	1	------
750	6	2	------	2	1	------	2	1	------	1	------	------
760	4	2	------	1	1	------	1	1	------	------	------	------
770	2	------	------	1	------	------	1	------	------	------	------	------
Σ=	109828	100000	35547	98041	100000	118103	96124	100000	124379	100078	100000	231410
x, y, z=	0. 44759	0. 40754	0. 14487	0. 31012	0. 31631	0. 37357	0. 29992	0. 31201	0. 38807	0. 23194	0. 23176	0. 53630

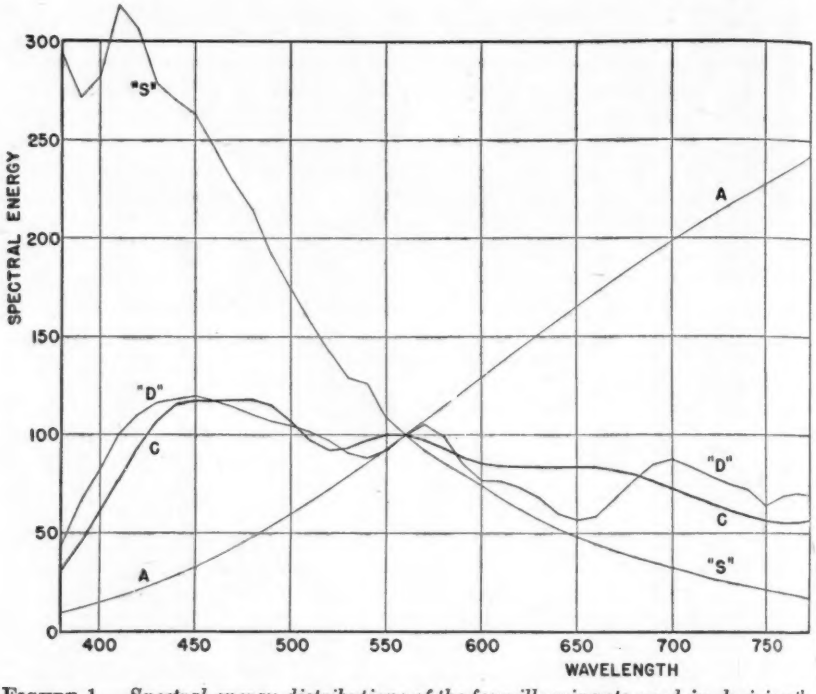

FIGURE 1.—*Spectral energy distributions of the four illuminants used in deriving the colorimetric data on the Munsell standards.*

ICI illuminant *A*, 2,842° K, representative of incandescent illuminants. ICI illuminant *C*, representative of average daylight. Illuminant "*D*", representative of lightly overcast sky. Illuminant "*S*", representative of "limit blue sky."

IV. COLORIMETRIC DATA

Values of X, Y, Z, x, and y for all of the samples and for the four illuminants, as explained above, together with the Munsell notations, $H\ V/C$ (hue, value, and chroma), and the Munsell painting number for each sample, are given in table 2. Values for the neutrals are at the end of the table. Values of z are omitted, since $z=1-x-y$.

Values of the trilinear coordinates, x and y, for ICI illuminant C, are plotted in figures 2 to 8 for Munsell values 2 to 8, respectively. The x and y values for ICI illuminant C, and therefore for magnesium oxide and for any other spectrally nonselective sample, are given in each diagram at $x=0.3101$, $y=0.3163$. Values of x and y for the Munsell samples obtained at the National Bureau of Standards are plotted as circled points. The data obtained by Glenn and Killian [8] at the Massachusetts Institute of Technology in 1935 are plotted as uncircled points for comparison with the present data. When the two points for a sample coincide, the combination is plotted, as a circled point with a short line attached; in many cases, to avoid confusion, the two points are joined by a fine line. Lines are drawn connecting all of the NBS points of constant chroma on each diagram, resulting in the spiderweb-like figures shown.

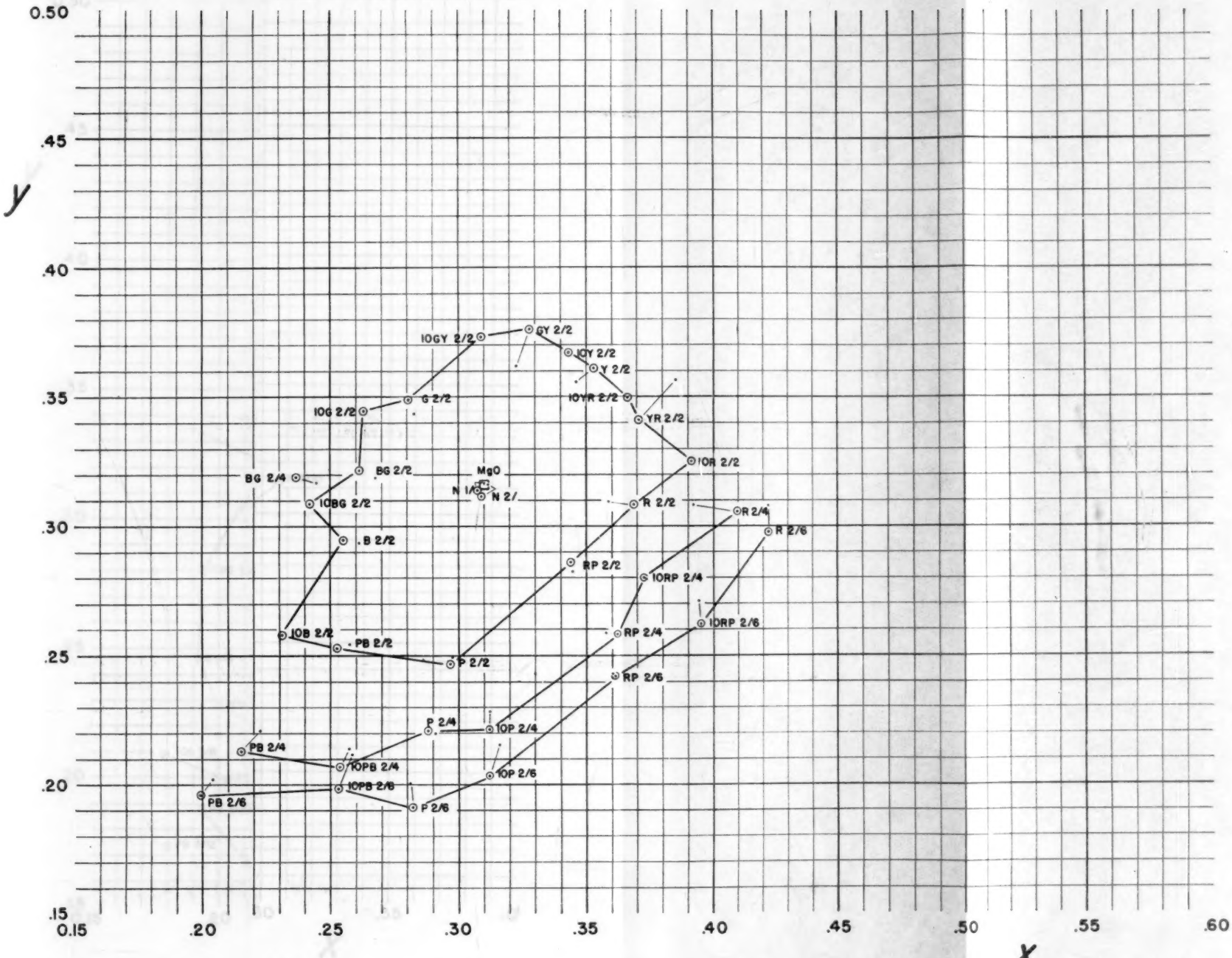

FIGURE 2.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 2/.*

538330—43 (Face p. 60)

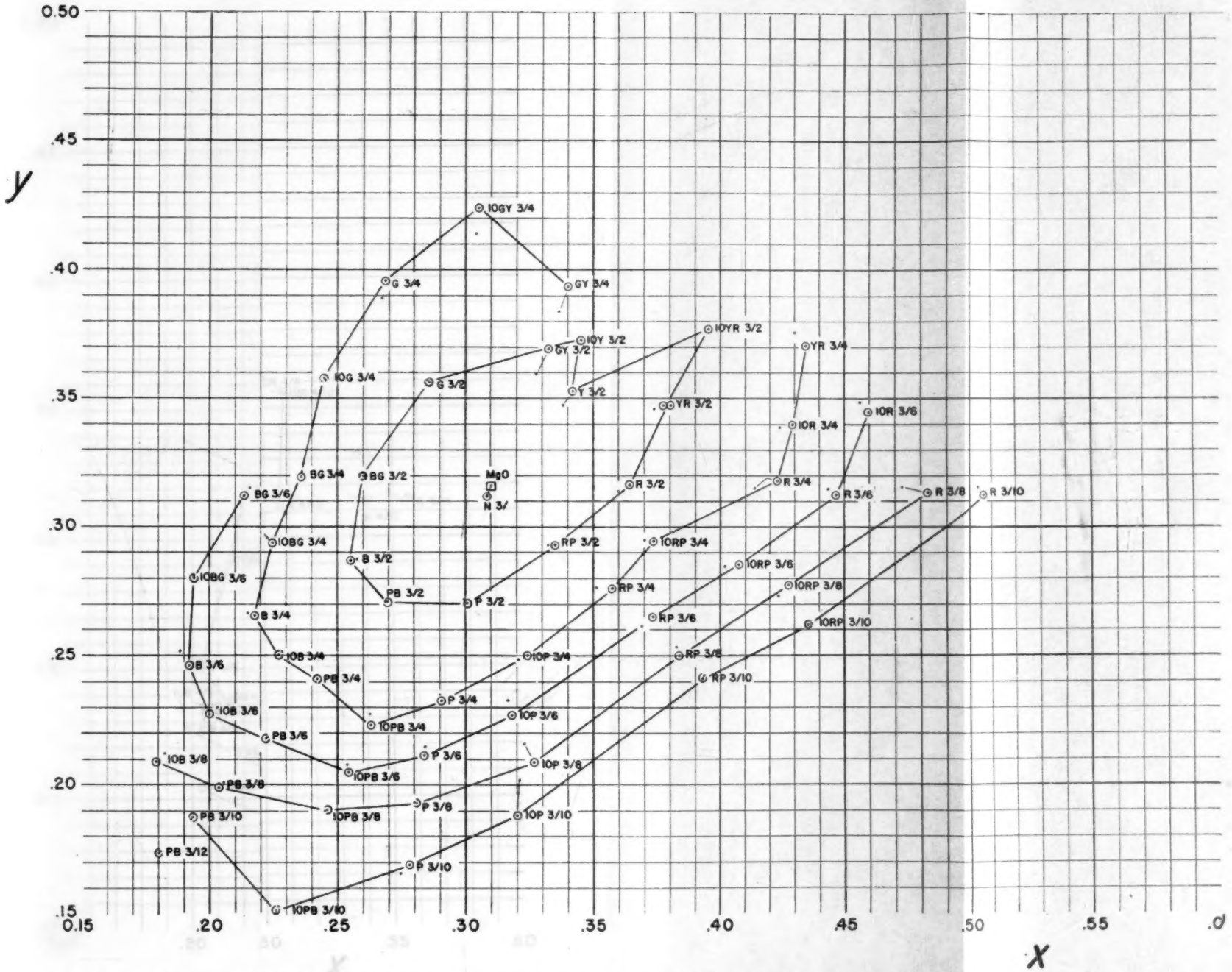

FIGURE 3.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 3/.*

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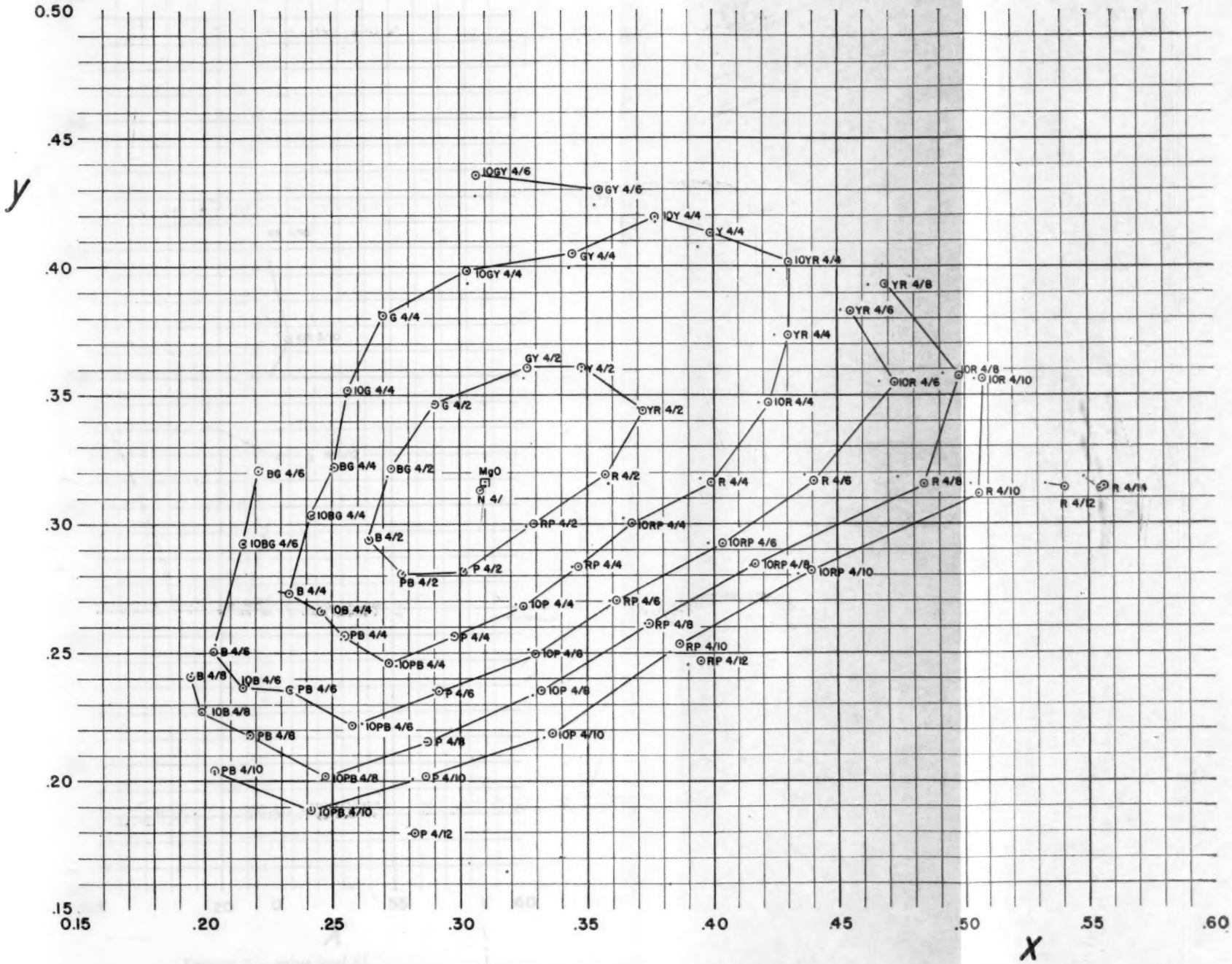

FIGURE 4.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 4/.*

538330—43 (Face p. 60)

0.30
.45
.40
.35
.30
.25
.20
.15
.35
.30
.25
.20
.15

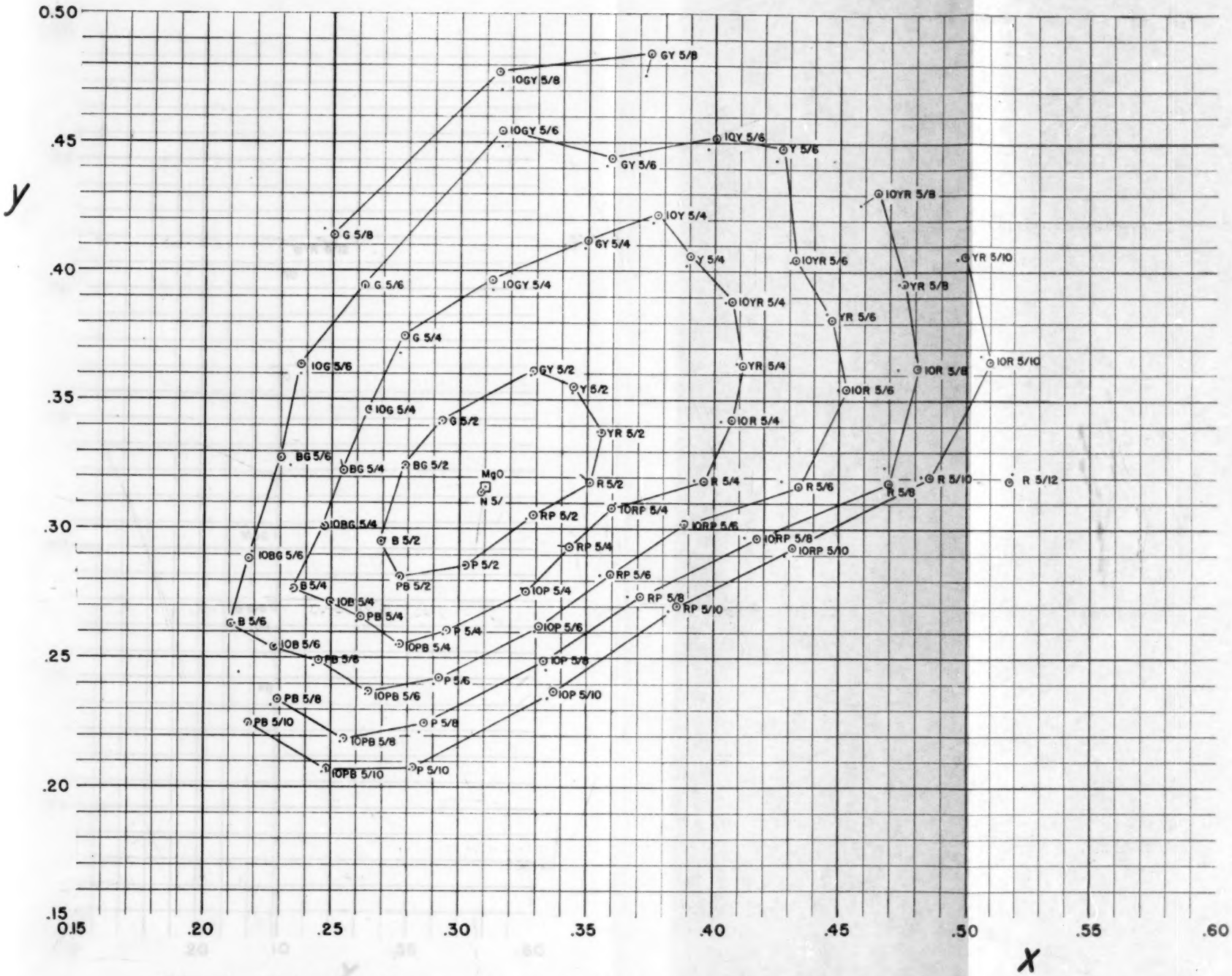

FIGURE 5.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 5/.*

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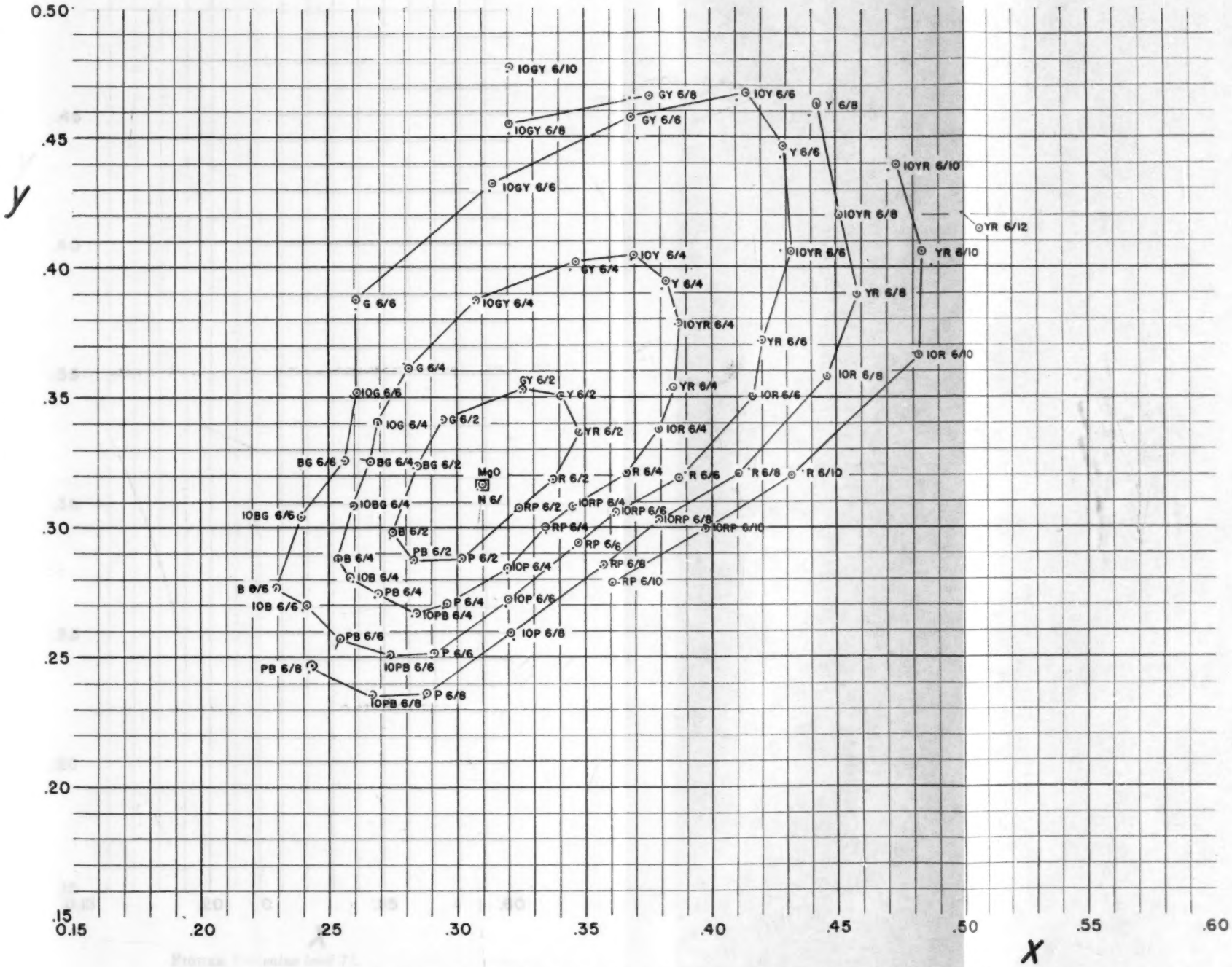

FIGURE 6.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 6/.*

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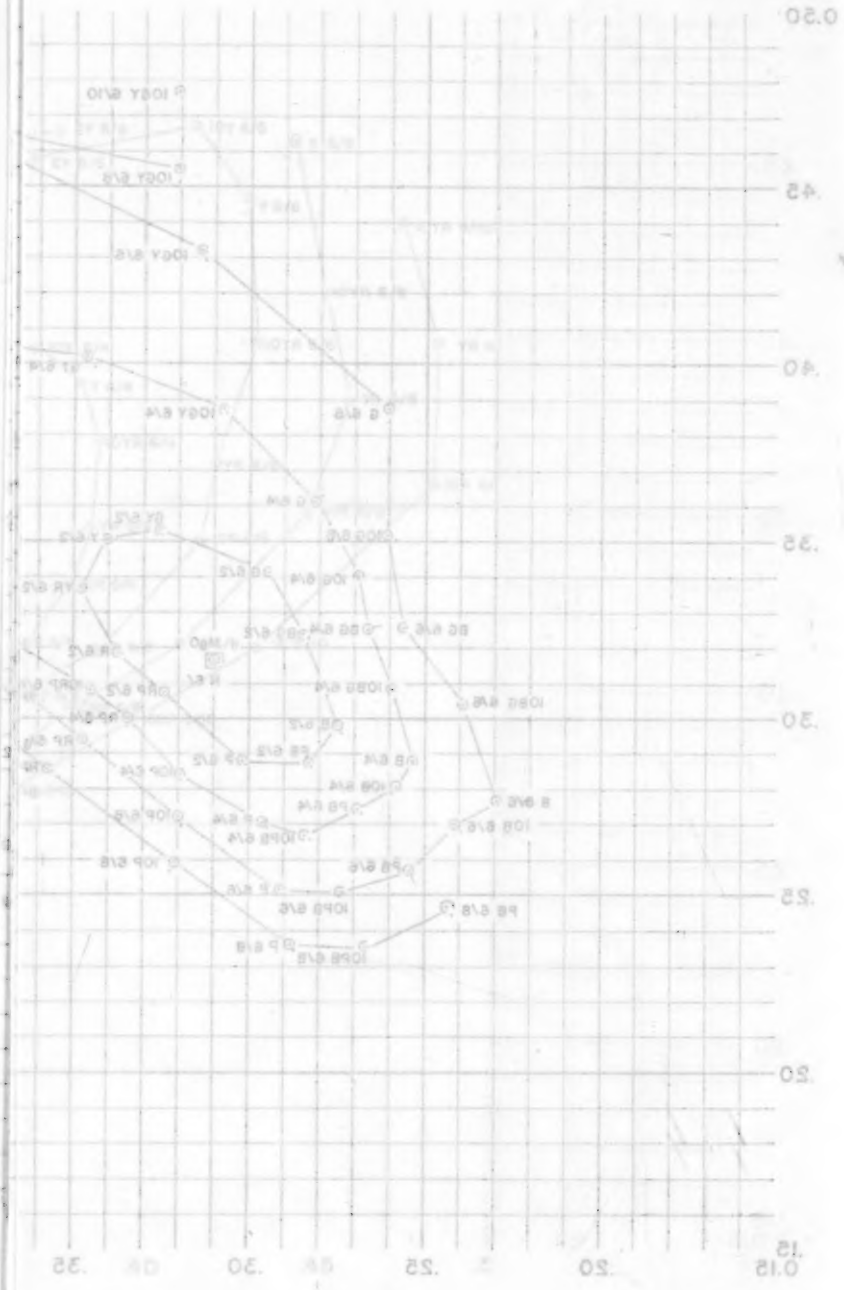

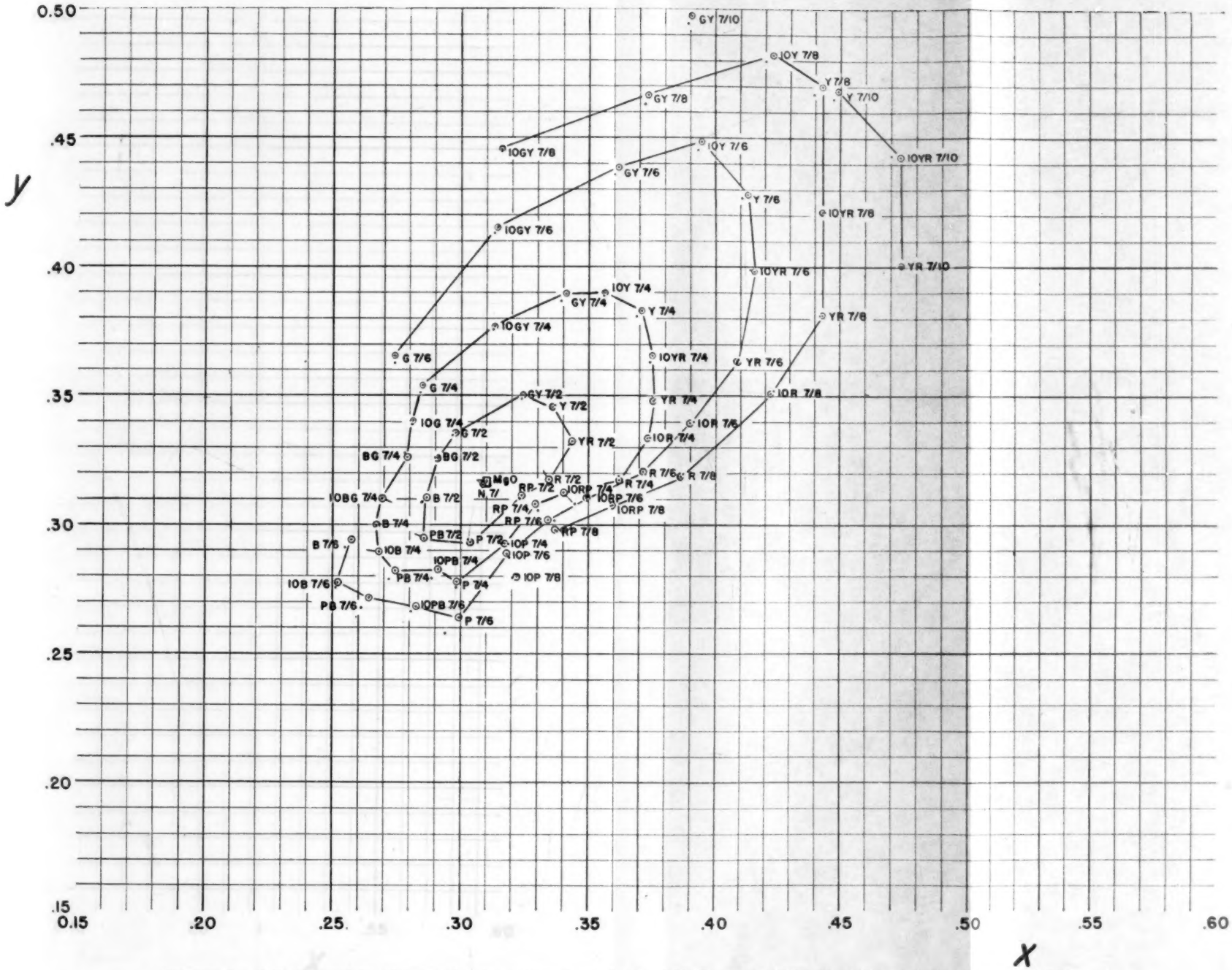

FIGURE 7.—*ICI chromaticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 7/.*

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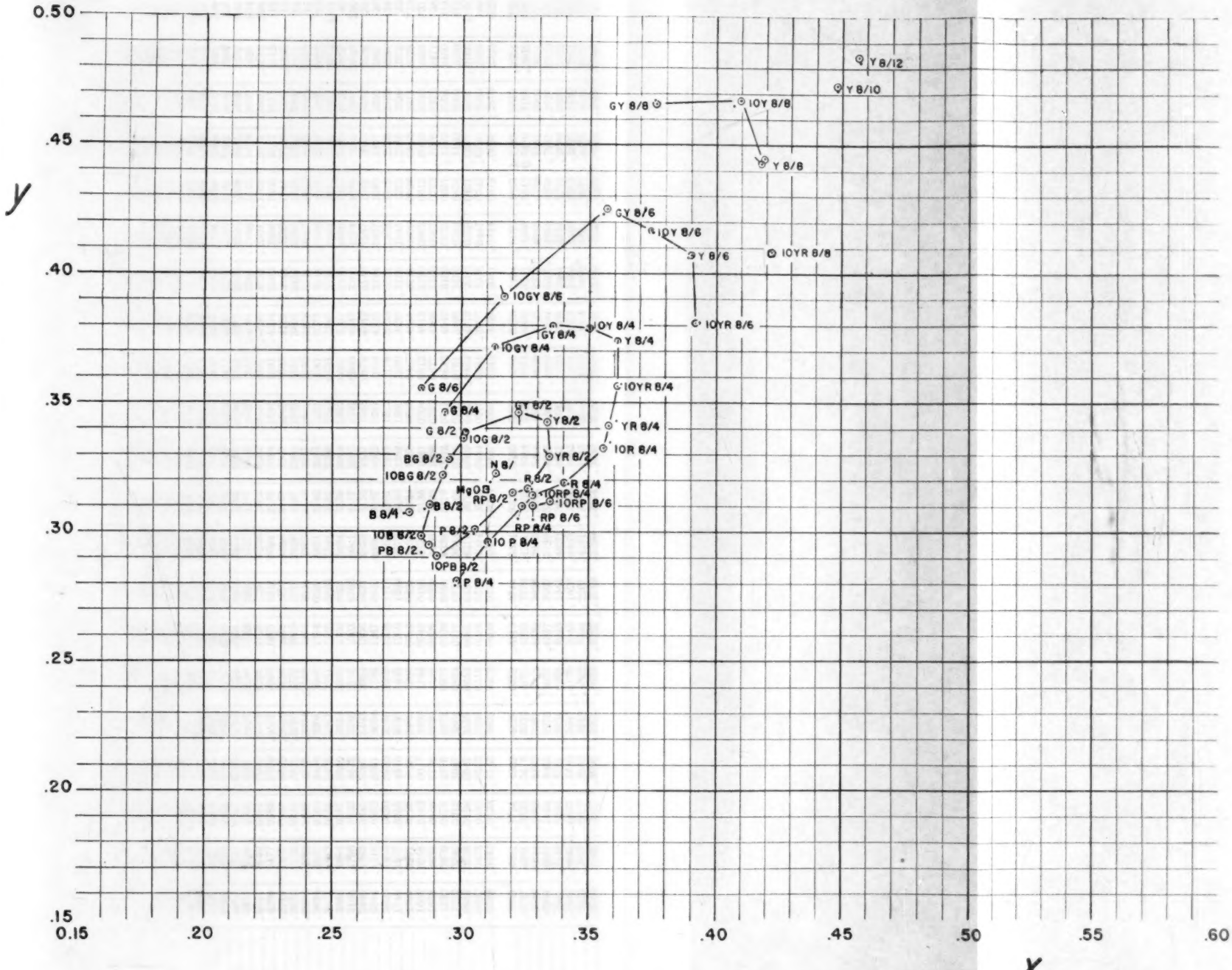

FIGURE 8.—*ICI chromiticity diagram showing values of x and y for ICI illuminant C for Munsell standards of value level 8/.*

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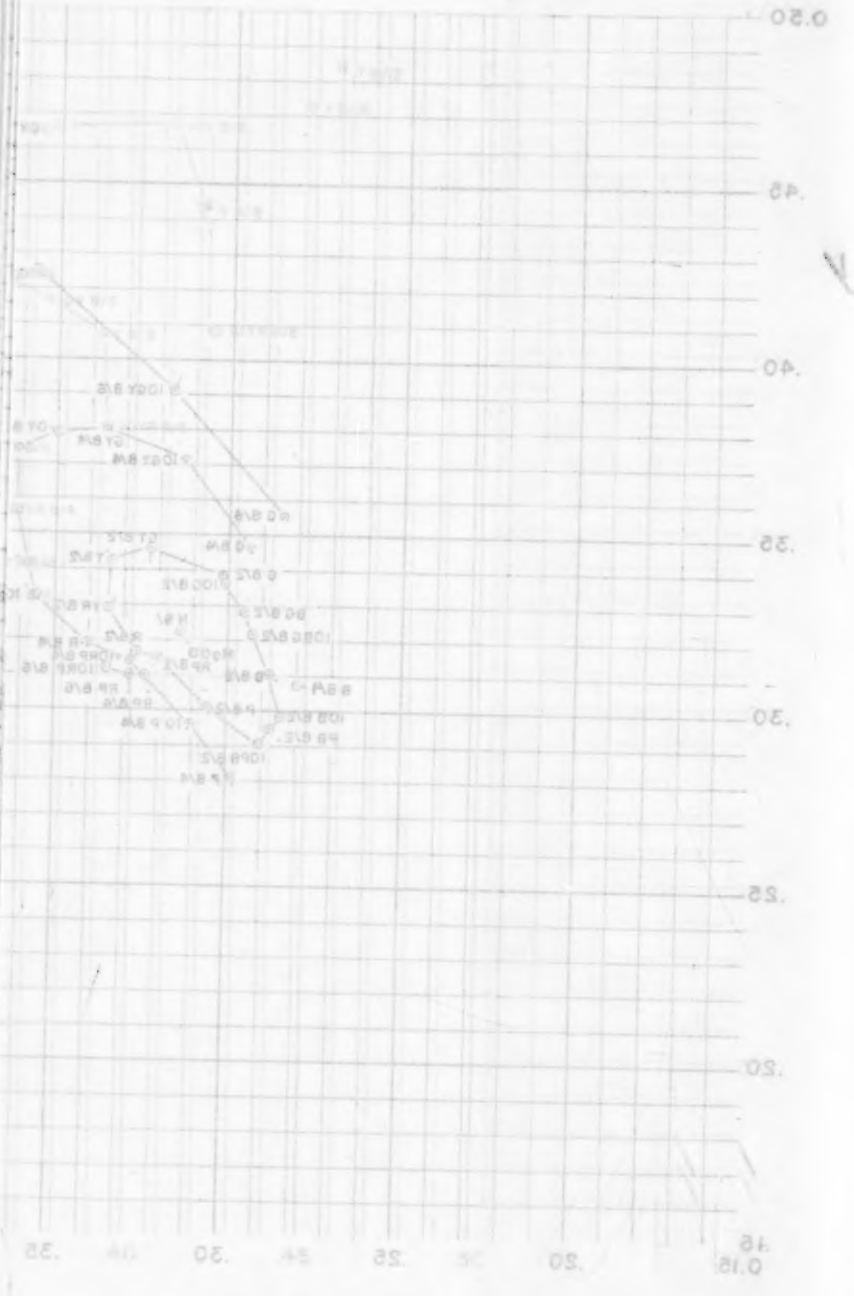

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell [illegible]*

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
R 8/4	0.7432	0.6147	0.1874	0.4809	0.3978	0.6261	0.5862	0.6264	0.3405	0.3188	0.6075	0.5832	0.6604	0.3282	0.3150	0.6044	0.5667	1.2317	0.2516	0.2358	1168
2	.7160	.6179	.2036	.4657	.4019	.6199	.6021	.6793	.3261	.3167	.6047	.6007	.7158	.3148	.3127	.6147	.5908	1.3338	.2421	.2327	342
7/8	.6564	.4769	.1168	.5251	.3815	.5144	.4244	.3936	.3861	.3185	.4909	.4178	.4159	.3706	.3154	.4614	.3903	0.7794	.2829	.2393	84
6	.6445	.4886	.1276	.5112	.3876	.5157	.4452	.4279	.3713	.3206	.4946	.4400	.4515	.3568	.3174	.4722	.4167	.8437	.2725	.2405	880
4	.6173	.4777	.1324	.5030	.3892	.5020	.4402	.4449	.3619	.3174	.4835	.4361	.4696	.3480	.3139	.4682	.4153	.8784	.2657	.2357	82
2	.5586	.4699	.1485	.4746	.3993	.4763	.4521	.4965	.3343	.3173	.4632	.4503	.5234	.3224	.3134	.4655	.4396	.9765	.2474	.2336	80
6/10	.5912	.3874	.0740	.5616	.3681	.4369	.3236	.2514	.4318	.3198	.4102	.3146	.2662	.4139	.3174	.3666	.2834	.5014	.3184	.2461	1156
8	.5461	.3758	.0804	.5448	.3749	.4142	.3236	.2717	.4103	.3205	.3920	.3167	.2873	.3936	.3180	.3579	.2902	.5396	.3013	.2443	338
6	.5121	.3721	.0903	.5255	.3818	.4013	.3308	.3053	.3868	.3189	.3831	.3259	.3229	.3713	.3158	.3598	.3036	.6064	.2834	.2391	728
4	.4673	.3611	.0958	.5056	.3907	.3787	.3313	.3228	.3667	.3208	.3649	.3284	.3411	.3528	.3175	.3510	.3106	.6393	.2698	.2388	726
2	.4007	.3345	.1037	.4776	.3987	.3396	.3205	.3464	.3374	.3184	.3298	.3190	.3651	.3253	.3146	.3295	.3108	.6805	.2496	.2353	332
5/12	.4675	.2596	.0293	.6181	.3432	.3149	.1938	.0995	.5177	.3187	.2858	.1825	.1055	.4981	.3181	.2346	.1545	.1989	.3990	.2628	894
10	.4651	.2732	.0381	.5990	.3519	.3226	.2128	.1291	.4855	.3202	.2962	.2030	.1367	.4659	.3192	.2501	.1760	.2572	.3660	.2576	729
8	.4099	.2477	.0388	.5886	.3557	.2900	.1963	.1315	.4694	.3178	.2684	.1884	.1391	.4504	.3162	.2306	.1650	.2612	.3511	.2512	18
6	.3713	.2407	.0465	.5639	.3656	.2736	.1998	.1574	.4338	.3167	.2567	.1940	.1663	.4161	.3144	.2289	.1744	.3120	.3200	.2438	16
4	.3074	.2183	.0510	.5331	.3785	.2379	.1915	.1715	.3959	.3187	.2264	.1881	.1810	.3801	.3159	.2100	.1743	.3386	.2905	.2411	14
2	.2604	.2084	.0610	.4315	.3934	.2158	.1954	.2041	.3507	.3176	.2087	.1941	.2150	.3378	.3141	.2047	.1865	.4007	.2585	.2356	12
4/14	.3644	.1881	.0163	.6406	.3308	.2360	.1334	.0550	.5561	.3143	.2104	.1231	.0581	.5373	.3144	.1666	.1015	.1089	.4420	.2692	42R
14	.3653	.1882	.0166	.6408	.3301	.2361	.1335	.0563	.5544	.3135	.2100	.1231	.0595	.5348	.3135	.1664	.1015	.1119	.4382	.2673	1151
12	.3580	.1884	.0187	.6336	.3333	.2341	.1360	.0631	.5404	.3139	.2094	.1262	.0667	.5207	.3136	.1680	.1055	.1250	.4216	.2647	40
10	.3166	.1736	.0226	.6173	.3386	.2116	.1303	.0762	.5061	.3117	.1905	.1221	.0805	.4847	.3106	.1574	.1050	.1510	.3807	.2540	730
8	.2890	.1683	.0245	.5997	.3493	.2004	.1306	.0831	.4839	.3155	.1837	.1243	.0878	.4642	.3140	.1557	.1082	.1648	.3632	.2524	38 R
6	.2374	.1516	.0282	.5691	.3634	.1734	.1247	.0955	.4406	.3167	.1622	.1207	.1010	.4225	.3144	.1437	.1080	.1897	.3255	.2447	36R
4	.2023	.1418	.0330	.5365	.3760	.1559	.1236	.1115	.3987	.3161	.1481	.1212	.1178	.3825	.3131	.1373	.1119	.2210	.2920	.2381	34
2	.1748	.1377	.0388	.4976	.3920	.1433	.1280	.1299	.3572	.3190	.1383	.1269	.1369	.3439	.3156	.1345	.1213	.2555	.2630	.2373	32
3/10	.1799	.0991	.0126	.6168	.3400	.1206	.0744	.0431	.5065	.3124	.1088	.0698	.0457	.4851	.3113	.0900	.0596	.0862	.3818	.2528	732
8	.1662	.0958	.0142	.6017	.3468	.1143	.0743	.0484	.4824	.3135	.1041	.0704	.0512	.4613	.3120	.0882	.0613	.0963	.3588	.2495	731
6	.1653	.1021	.0188	.5776	.3569	.1185	.0829	.0639	.4466	.3125	.1098	.0797	.0677	.4268	.3100	.0967	.0712	.1276	.3272	.2411	738
4	.1320	.0870	.0178	.5576	.3673	.0977	.0734	.0599	.4228	.3179	.0914	.0713	.0632	.4047	.3155	.0823	.0651	.1183	.3098	.2450	314
2	.1022	.0780	.0216	.5067	.3864	.0824	.0716	.0722	.3643	.3164	.0790	.0707	.0761	.3499	.3130	.0762	.0674	.1420	.2669	.2360	312
2/6	.0885	.0558	.0124	.5647	.3559	.0653	.0461	.0434	.4219	.2977	.0609	.0445	.0462	.4019	.2934	.0558	.0401	.0877	.3037	.2186	1124
4	.0687	.0453	.0105	.5522	.3639	.0516	.0384	.0359	.4094	.3053	.0484	.0373	.0381	.3907	.3015	.0447	.0342	.0720	.2960	.2268	860
2	.0598	.0442	.0123	.5143	.3803	.0479	.0400	.0420	.3686	.3083	.0457	.0394	.0445	.3529	.3041	.0444	.0373	.0839	.2679	.2253	322
10R 8/4	.7899	.6491	.1724	.4902	.4028	.6539	.6115	.5744	.3554	.3324	.6339	.6087	.6056	.3430	.3294	.6136	.5823	1.1293	.2639	.2504	722
7/8	.6797	.4921	.0333	.5416	.3921	.5161	.4290	.2776	.4221	.3509	.4914	.4228	.2934	.4069	.3501	.4366	.3842	0.5489	.3188	.2805	882
6	.6605	.5044	.1085	.5187	.3961	.5222	.4550	.3635	.3895	.3394	.5018	.4508	.3839	.3755	.3373	.4646	.4180	.7181	.2903	.2611	714
4	.6288	.4937	.1194	.5064	.3976	.5075	.4537	.3997	.3729	.3334	.4893	.4502	.4219	.3594	.3307	.4634	.4236	.7881	.2767	.2529	667
6/10	.5891	.3948	.0404	.5751	.3854	.4268	.3243	.1345	.4820	.3662	.4031	.3181	.1422	.4668	.3684	.3354	.2732	.2664	.3833	.3122	583
8	.5585	.3931	.0547	.5550	.3907	.4170	.3345	.1828	.4463	.3580	.3965	.3295	.1934	.4313	.3584	.3426	.2913	.3623	.3439	.2924	585
6	.5243	.3853	.0677	.5365	.3942	.4022	.3384	.2261	.4161	.3501	.3843	.3344	.2388	.4014	.3492	.3434	.3037	.4465	.3140	.2777	733
4	.4556	.3569	.0817	.5096	.3991	.3663	.3262	.2743	.3789	.3374	.3535	.3239	.2899	.3654	.3349	.3322	.3026	.5431	.2820	.2569	587

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*—Continued

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
10R 6/4	.4710	.3673	.0802	.5128	.3999	.3755	.3347	.2667	.3844	.3426	.3621	.3326	.2814	.3710	.3407	.3360	.3095	.5254	.2870	.2643	1860
5/10	.3965	.2488	.0208	.5953	.3735	.2761	.1977	.0678	.5098	.3650	.2562	.1912	.0716	.4937	.3684	.2078	.1633	.1331	.4121	.3239	590
8	.3990	.2653	.0284	.5761	.3830	.2884	.2175	.0943	.4806	.3624	.2719	.2130	.0996	.4652	.3644	.2266	.1833	.1861	.3802	.3076	734
6	.3591	.2465	.0339	.5615	.3855	.2650	.2073	.1130	.4527	.3542	.2508	.2035	.1194	.4371	.3547	.2155	.1796	.2234	.3485	.2904	588
4	.3102	.2297	.0442	.5311	.3932	.2409	.2028	.1484	.4089	.3425	.2309	.2007	.1568	.3924	.3411	.2093	.1829	.2937	.3052	.2666	580
4/10	.3031	.1891	.0172	.5950	.3712	.2127	.1493	.0571	.5075	.3563	.1981	.1446	.0603	.4916	.3587	.1619	.1225	.1126	.4078	.3086	761
8	.2811	.1786	.0176	.5890	.3742	.1989	.1428	.0580	.4976	.3573	.1858	.1387	.0611	.4818	.3598	.1528	.1183	.1138	.3970	.3075	747
6	.2627	.1734	.0209	.5749	.3794	.1897	.1424	.0693	.4726	.3549	.1783	.1392	.0730	.4567	.3563	.1500	.1210	.1359	.3686	.2974	735
4	.2158	.1550	.0269	.5426	.3898	.1638	.1348	.0896	.4220	.3472	.1559	.1328	.0946	.4067	.3464	.1385	.1203	.1766	.3182	.2763	591
3/6	.1440	.0951	.0133	.5705	.3766	.1047	.0786	.0446	.4594	.3448	.0984	.0766	.0472	.4428	.3449	.0844	.0674	.0884	.3513	.2808	736
4	.1250	.0866	.0151	.5515	.3820	.0935	.0741	.0502	.4291	.3402	.0884	.0726	.0530	.4129	.3393	.0782	.0658	.0990	.3219	.2706	595
2/2	.0711	.0515	.0119	.5288	.3828	.0551	.0458	.0399	.3913	.3252	.0523	.0450	.0421	.3753	.3226	.0486	.0420	.0788	.2870	.2478	915
YR 8/4	.7567	.6314	.1590	.4891	.4081	.6255	.5970	.5271	.3575	.3412	.6068	.5950	.5557	.3453	.3386	.5829	.5690	1.0348	.2666	.2602	385
2	.7041	.6096	.1824	.4706	.4075	.6007	.5917	.6039	.3344	.3294	.5849	.5904	.6358	.3230	.3260	.5825	.5777	1.1813	.2488	.2467	384
7/10	.6736	.4888	.0415	.5595	.4060	.4941	.4178	.1309	.4738	.4007	.4696	.4129	.1378	.4603	.4047	.3870	.3629	0.2534	.3857	.3617	867
8	.6729	.4991	.0629	.5449	.4042	.5054	.4356	.2002	.4429	.3817	.4826	.4314	.2097	.4295	.3839	.4108	.3860	.3843	.3478	.3268	740
6	.6086	.4657	.0808	.5269	.4032	.4703	.4180	.2635	.4084	.3629	.4504	.4140	.2776	.3944	.3626	.4022	.3816	.5151	.3097	.2938	739
6	.6194	.4746	.0822	.5266	.4035	.4787	.4262	.2676	.4083	.3635	.4584	.4223	.2817	.3944	.3633	.4090	.3894	.5219	.3098	.2949	739
4	.5867	.4724	.1052	.5039	.4057	.4723	.4376	.3484	.3753	.3478	.4556	.4349	.3674	.3622	.3457	.4268	.4110	.6840	.2805	.2701	154
2	.5362	.4561	.1287	.4783	.4069	.4521	.4375	.4275	.3433	.3322	.4396	.4362	.4505	.3315	.3289	.4325	.4229	.8385	.2553	.2497	152
6/12	.5511	.3886	.0203	.5740	.4048	.3962	.3241	.0617	.5067	.4145	.3757	.3199	.0647	.4941	.4208	.2994	.2742	.1175	.4333	.3967	383
10	.5465	.3945	.0290	.5634	.4067	.3989	.3350	.0915	.4833	.4059	.3787	.3307	.0963	.4700	.4105	.3095	.2886	.1771	.3992	.3723	745
8	.5163	.3798	.0394	.5519	.4060	.3849	.3274	.1278	.4582	.3897	.3677	.3243	.1348	.4448	.3922	.3092	.2862	.2500	.3657	.3385	896
6	.4721	.3620	.0549	.5477	.4072	.3640	.3221	.1804	.4201	.3717	.3496	.3199	.1904	.4066	.3720	.3077	.2902	.3543	.3232	.3048	379
4	.4231	.3390	.0693	.5089	.4077	.3389	.3113	.2300	.3850	.3537	.3275	.3099	.2428	.3721	.3521	.3024	.2886	.4528	.2897	.2765	377
2	.3834	.3266	.0883	.4803	.4091	.3225	.3124	.2936	.3473	.3364	.3139	.3117	.3097	.3356	.3333	.3066	.3003	.5772	.2590	.2536	375
5/10	.3490	.2462	.0155	.5715	.4032	.2528	.2056	.0482	.4990	.4058	.2402	.2032	.0506	.4862	.4113	.1933	.1740	.0924	.4205	.3785	868
8	.3392	.2450	.0215	.5601	.4044	.2503	.2081	.0680	.4755	.3953	.2393	.2063	.0714	.4628	.3991	.1969	.1790	.1311	.3883	.3531	748
6	.3095	.2282	.0275	.5477	.4037	.2320	.1981	.0992	.4468	.3815	.2212	.1960	.0939	.4328	.3834	.1887	.1750	.1741	.3509	.3254	741
4	.2787	.2131	.0359	.5282	.4039	.2156	.1905	.1181	.4113	.3633	.2067	.1888	.1246	.3975	.3630	.1844	.1730	.2318	.3129	.2937	144
2	.2443	.2037	.0527	.4880	.4069	.2029	.1927	.1755	.3553	.3374	.1969	.1920	.1852	.3430	.3344	.1905	.1839	.3457	.2646	.2554	142
4/8	.2159	.1585	.0147	.5549	.4074	.1614	.1354	.0474	.4689	.3935	.1551	.1348	.0498	.4566	.3968	.1286	.1166	.0919	.3816	.3458	902
6	.2320	.1678	.0191	.5539	.4006	.1717	.1445	.0610	.4552	.3830	.1630	.1425	.0642	.4409	.3855	.1376	.1271	.1183	.3592	.3319	875
4	.2167	.1610	.0229	.5410	.4018	.1632	.1416	.0744	.4303	.3735	.1551	.1397	.0784	.4157	.3743	.1350	.1276	.1452	.3310	.3129	987
2	.1685	.1366	.0314	.5008	.4059	.1370	.1268	.1044	.3720	.3444	.1326	.1262	.1100	.3594	.3423	.1248	.1189	.2049	.2783	.2650	584
3/4	.1212	.0897	.0127	.5422	.4012	.0918	.0783	.0412	.4344	.3705	.0878	.0775	.0434	.4206	.3714	.0761	.0697	.0805	.3362	.3079	988
2	.0955	.0759	.0165	.5084	.4038	.0764	.0698	.0548	.3803	.3473	.0737	.0693	.0578	.3669	.3453	.0687	.0653	.1078	.2841	.2700	989
2	.0949	.0759	.0168	.5058	.4045	.0761	.0701	.0556	.3774	.3473	.0734	.0696	.0585	.3643	.3454	.0685	.0657	.1088	.2820	.2702	1201
2/2	.0592	.0473	.0113	.5026	.4017	.0477	.0439	.0371	.3707	.3411	.0461	.0436	.0391	.3576	.3387	.0434	.0415	.0726	.2757	.2634	857

10YR 8/8	.8119	.6627	.0804	.5221	.4262	.6249	.6049	.2514	.4219	.4084	.6004	.6019	.2644	.4094	.4104	.5177	.5546	.4849	.3325	.3562	1595
6	.7891	.6583	.1118	.5061	.4222	.6282	.6114	.3648	.3916	.3811	.6067	.6090	.3846	.3791	.3806	.5494	.5700	.7139	.2997	.3109	593
4	.7339	.6279	.1436	.4875	.4171	.6048	.5972	.4732	.3610	.3565	.5868	.5955	.4989	.3490	.3542	.5575	.5706	.9277	.2712	.2775	589
7/10	.5991	.4740	.0264	.5449	.4311	.4470	.4179	.0793	.4734	.4426	.4288	.4157	.0834	.4621	.4480	.3487	.3676	.1515	.4018	.4237	603
8	.6084	.4895	.0450	.5323	.4283	.4621	.4397	.1411	.4431	.4216	.4434	.4371	.1487	.4308	.4247	.3738	.3958	.2733	.3584	.3795	744
6	.5817	.4763	.0628	.5190	.4250	.4533	.4347	.2029	.4155	.3985	.4371	.4330	.2141	.4031	.3994	.3827	.3979	.3971	.3250	.3378	601
4	.5815	.4899	.0994	.4967	.4184	.4716	.4602	.3268	.3747	.3656	.4567	.4588	.3448	.3624	.3640	.4251	.4346	.6417	.2831	.2895	600
6/10	.5214	.4084	.0245	.5464	.4280	.3873	.3595	.0711	.4735	.4396	.3713	.3579	.0742	.4621	.4455	.3006	.3172	.1322	.4008	.4230	1422
8	.4912	.3887	.0342	.5374	.4252	.3713	.3457	.1060	.4512	.4201	.3569	.3443	.1114	.4393	.4237	.2978	.3084	.2037	.3677	.3808	749
6	.4721	.3770	.0425	.5295	.4229	.3611	.3392	.1352	.4322	.4059	.3468	.3373	.1425	.4196	.4081	.2971	.3068	.2629	.3428	.3540	750
4	.4011	.3376	.0596	.5024	.4229	.3218	.3149	.1952	.3869	.3786	.3116	.3141	.2060	.3747	.3777	.2843	.2946	.3833	.2955	.3062	605
5/8	.2999	.2379	.0166	.5409	.4292	.2266	.2099	.0507	.4651	.4308	.2191	.2099	.0532	.4544	.4353	.1795	.1844	.0966	.3898	.4005	612
6	.2966	.2360	.0269	.5301	.4218	.2267	.2121	.0853	.4325	.4047	.2177	.2109	.0898	.4199	.4069	.1864	.1918	.1654	.3428	.3529	743
4	.2719	.2236	.0331	.5143	.4230	.2145	.2048	.1077	.4070	.3885	.2078	.2045	.1137	.3951	.3888	.1841	.1878	.2111	.3159	.3221	610
4/4	.1819	.1467	.0171	.5262	.4243	.1413	.1318	.0548	.4309	.4019	.1372	.1320	.0577	.4198	.4037	.1174	.1181	.1063	.3435	.3455	613
3/2	.0915	.0755	.0130	.5084	.4195	.0730	.0696	.0421	.3953	.3768	.0709	.0696	.0443	.3836	.3765	.0638	.0644	.0820	.3035	.3064	933
2/2	.0650	.0537	.0124	.4954	.4098	.0529	.0505	.0410	.3664	.3496	.0511	.0502	.0432	.3538	.3474	.0483	.0479	.0803	.2739	.2711	934
Y 8/12	.7051	.6199	.0257	.5220	.4589	.5374	.5706	.0697	.4563	.4845	.5178	.5685	.0731	.4466	.4903	.4218	.5167	.1288	.3952	.4841	62
12	.6855	.5924	.0285	.5247	.4535	.5178	.5443	.0744	.4556	.4789	.4978	.5427	.0774	.4453	.4855	.4038	.4958	.1333	.3909	.4800	2054
10	.7190	.6269	.0343	.5209	.4542	.5474	.5784	.0959	.4481	.4734	.5265	.5760	.1004	.4377	.4789	.4328	.5281	.1780	.3800	.4638	857a
8	.7015	.6109	.0625	.5102	.4443	.5406	.5732	.1804	.4177	.4429	.5205	.5722	.1883	.4093	.4467	.4409	.5373	.3358	.3355	.4089	852
8	.7093	.6177	.0617	.5108	.4448	.5457	.5794	.1770	.4191	.4450	.5253	.5783	.1847	.4078	.4489	.4438	.5429	.3284	.3375	.4128	852
6	.6938	.6110	.0904	.4973	.4379	.5539	.5784	.2874	.3902	.4074	.5357	.5769	.3025	.3785	.4077	.4798	.5459	.5572	.3031	.3449	58
6	.6633	.5858	.0901	.4953	.4374	.5305	.5566	.2834	.3871	.4061	.5136	.5558	.2978	.3757	.4065	.4602	.5273	.5459	.3001	.3439	1221
4	.6626	.5871	.1239	.4824	.4274	.5455	.5651	.4007	.3609	.3739	.5293	.5640	.4214	.3494	.3724	.4974	.5440	.7781	.2734	.2990	56
2	.6357	.5680	.1610	.4558	.4162	.5427	.5570	.5265	.3337	.3425	.5287	.5563	.5533	.3227	.3395	.5224	.5469	1.0233	.2496	.2614	54
7/10	.5248	.4558	.0258	.5215	.4530	.4018	.4189	.0738	.4492	.4683	.3881	.4183	.0775	.4391	.4733	.3192	.3802	0.1387	.3809	.4536	904
8	.5161	.4557	.0272	.5166	.4562	.3975	.4220	.0786	.4426	.4698	.3842	.4211	.0825	.4327	.4743	.3180	.3848	.1482	.3737	.4522	70
6	.5124	.4455	.0500	.5084	.4420	.4014	.4158	.1549	.4129	.4278	.3882	.4153	.1628	.4018	.4298	.3354	.3861	.2974	.3292	.3789	390
4	.5079	.4479	.0856	.4877	.4301	.4143	.4282	.2756	.3706	.3830	.4022	.4278	.2901	.3591	.3820	.3720	.4091	.5357	.2825	.3107	388
2	.5054	.4546	.1250	.4658	.4190	.4326	.4454	.4113	.3356	.3454	.4225	.4453	.4334	.3247	.3422	.4172	.4361	.8053	.2515	.2630	387
6/8	.3880	.3410	.0220	.5167	.4540	.3009	.3145	.0649	.4423	.4623	.2922	.3151	.0681	.4327	.4665	.2424	.2858	.1228	.3723	.4390	903
8	.3994	.3513	.0224	.5166	.4514	.3095	.3242	.0658	.4425	.4635	.3006	.3247	.0690	.4329	.4678	.2490	.2948	.1240	.3729	.4414	903
6	.3954	.3429	.0298	.5148	.4465	.3055	.3179	.0888	.4289	.4464	.2951	.3177	.0933	.4180	.4499	.2487	.2932	.1686	.3501	.4126	1212
4	.3715	.3266	.0548	.4934	.4338	.3000	.3100	.1755	.3819	.3946	.2913	.3099	.1848	.3706	.3943	.2643	.2935	.3410	.2941	.3265	392
2	.3675	.3292	.0862	.4694	.4205	.3121	.3211	.2828	.3407	.3505	.3045	.3210	.2980	.3297	.3476	.2977	.3131	.5534	.2557	.2690	391
5/6	.2406	.2136	.0180	.5095	.4523	.1893	.1984	.0554	.4271	.4479	.1846	.1993	.0583	.4175	.4506	.1561	.1815	.1066	.3514	.4086	947
4	.2372	.2101	.0314	.4956	.4389	.1910	.1987	.0998	.3902	.4059	.1860	.1991	.1051	.3794	.4061	.1662	.1867	.1936	.3041	.3417	394
2	.2436	.2177	.0549	.4720	.4217	.2057	.2117	.1795	.3446	.3548	.2005	.2116	.1889	.3336	.3522	.1943	.2059	.3501	.2590	.2744	395
4/4	.1495	.1322	.0180	.4989	.4411	.1200	.1243	.0566	.3988	.4131	.1172	.1247	.0594	.3888	.4140	.1031	.1156	.1089	.3146	.3529	412
2	.1628	.1461	.0351	.4733	.4247	.1369	.1419	.1145	.3481	.3609	.1335	.1419	.1205	.3371	.3584	.1284	.1377	.2233	.2624	.2813	397
3/2	.0788	.0707	.0183	.4698	.4213	.0668	.0689	.0596	.3419	.3529	.0652	.0689	.0627	.3312	.3502	.0634	.0671	.1161	.2570	.2723	398
2/2	.0557	.0489	.0117	.4793	.4204	.0461	.0472	.0374	.3528	.3610	.0449	.0472	.0392	.3420	.3593	.0425	.0456	.0721	.2653	.2849	858
10Y 8/8	.6737	.6292	.0563	.4956	.4629	.5283	.6035	.1587	.4094	.4676	.5113	.6037	.1656	.3993	.4714	.4340	.5727	.2932	.3339	.4406	599
6	.6619	.6171	.0971	.4810	.4485	.5380	.5996	.3002	.3742	.4170	.5230	.6001	.3150	.3637	.4173	.4725	.5781	.5747	.2907	.3557	581
4	.6590	.6119	.1333	.4693	.4358	.5541	.5997	.4294	.3500	.3788	.5410	.6004	.4515	.3396	.3769	.5143	.5839	.8330	.2663	.3023	578
7/8	.4940	.4662	.0316	.4980	.4701	.3894	.4440	.0872	.4230	.4823	.3792	.4454	.0910	.4141	.4864	.3169	.4149	.1600	.3553	.4653	862
6	.4723	.4497	.0493	.4863	.4629	.3809	.4334	.1518	.3943	.4486	.3712	.4341	.1598	.3846	.4498	.3255	.4109	.2920	.3165	.3995	604
4	.4832	.4541	.0895	.4706	.4422	.4055	.4439	.2894	.3531	.3898	.3966	.4449	.3050	.3459	.3880	.3733	.4296	.5648	.2729	.3141	608
6/6	.3665	.3437	.0291	.4957	.4649	.2909	.3278	.0830	.4145	.4671	.2838	.3292	.0866	.4056	.4706	.2398	.3072	.1536	.3423	.4385	617
4	.3671	.3393	.0583	.4800	.4437	.3007	.3291	.1842	.3694	.4043	.2925	.3294	.1937	.3587	.4038	.2681	.3170	.3559	.2849	.3369	615

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*—Continued

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
10Y 5/6	.2480	.2345	.0245	.4892	.4626	.1996	.2250	.0736	.4006	.4517	.1952	.2262	.0772	.3916	.4536	.1687	.2119	.1395	.3243	.4074	921
4	.2431	.2292	.0342	.4800	.4525	.1989	.2224	.1059	.3773	.4219	.1943	.2232	.1111	.3676	.4223	.1745	.2131	.2028	.2956	.3610	619
4/4	.1510	.1430	.0212	.4791	.4537	.1245	.1385	.0672	.3771	.4194	.1221	.1392	.0707	.3677	.4192	.1102	.1320	.1302	.2959	.3544	622
3/2	.0863	.0801	.0186	.4665	.4330	.0730	.0788	.0598	.3449	.3726	.0713	.0789	.0627	.3348	.3706	.0683	.0772	.1154	.2617	.2959	935
2/2	.0535	.0493	.0119	.4666	.4299	.0453	.0484	.0382	.3431	.3672	.0443	.0485	.0401	.3331	.3653	.0424	.0473	.0737	.2597	.2897	939
GY 8/8	.5094	.6050	.0728	.4693	.4737	.4857	.6022	.2046	.3758	.4659	.4731	.6045	.2130	.3666	.4684	.4139	.5903	.3753	.3000	.4279	432
6	.6046	.5941	.1017	.4650	.4568	.4977	.5929	.3048	.3567	.4249	.4847	.5944	.3186	.3468	.4253	.4422	.5858	.5741	.2760	.3657	433
4	.6293	.6035	.1436	.4572	.4354	.5341	.6041	.4522	.3358	.3799	.5215	.6052	.4740	.3258	.3781	.5015	.6008	.8675	.2546	.3050	434
2	.6266	.5850	.1727	.4527	.4226	.5451	.5851	.5606	.3224	.3461	.5333	.5856	.5891	.3123	.3429	.5341	.5833	1.0881	.2422	.2645	435
7/10	.4121	.4295	.0355	.4699	.4897	.3344	.4263	.0961	.3903	.4975	.3270	.4282	.1002	.3822	.5006	.2800	.4128	0.1749	.3227	.4757	908
8	.4413	.4524	.0540	.4657	.4774	.3606	.4512	.1555	.3728	.4665	.3519	.4529	.1624	.3639	.4683	.3104	.4419	.2892	.2981	.4243	442
6	.4507	.4518	.0689	.4640	.4651	.3716	.4509	.2057	.3614	.4386	.3625	.4524	.2151	.3519	.4393	.3279	.4439	.3874	.2829	.3829	441
4	.4523	.4372	.0964	.4588	.4434	.3822	.4369	.3023	.3408	.3896	.3733	.4379	.3170	.3309	.3882	.3552	.4330	.5796	.2597	.3166	439
2	.4539	.4260	.1224	.4528	.4250	.3944	.4261	.3970	.3239	.3500	.3860	.4266	.4173	.3138	.3469	.3851	.4244	.7711	.2436	.2685	437
6/8	.3183	.3257	.0380	.4667	.4776	.2612	.3238	.1104	.3756	.4657	.2554	.3253	.1153	.3669	.4674	.2251	.3154	.2059	.3016	.4226	715
6	.3113	.3210	.0403	.4629	.4772	.2576	.3200	.1216	.3685	.4577	.2519	.3211	.1277	.3596	.4582	.2261	.3126	.2317	.2935	.4058	446
4	.3296	.3223	.0644	.4601	.4500	.2772	.3217	.2013	.3464	.4020	.2708	.3225	.2113	.3366	.4009	.2549	.3178	.3865	.2657	.3313	445
2	.3443	.3247	.0906	.4533	.4274	.2991	.3245	.2949	.3257	.3533	.2928	.3249	.3104	.3155	.3501	.2917	.3226	.5747	.2453	.2714	444
5/8	.1948	.2096	.0208	.4582	.4929	.1621	.2100	.0612	.3742	.4846	.1593	.2110	.0643	.3666	.4856	.1405	.2041	.1158	.3053	.4432	450
6	.1990	.2062	.0298	.4575	.4741	.1669	.2066	.0915	.3589	.4443	.1636	.2075	.0962	.3501	.4440	.1495	.2028	.1757	.2832	.3842	720
4	.2078	.2074	.0379	.4587	.4576	.1756	.2070	.1196	.3497	.4121	.1720	.2077	.1259	.3403	.4108	.1613	.2035	.2314	.2705	.3414	449
2	.2201	.2098	.0555	.4535	.4323	.1908	.2097	.1804	.3284	.3611	.1868	.2101	.1900	.3184	.3580	.1849	.2083	.3520	.2481	.2795	448
4/6	.1219	.1248	.0202	.4568	.4677	.1031	.1249	.0626	.3547	.4298	.1013	.1256	.0658	.3462	.4292	.0934	.1224	.1202	.2780	.3642	907
4	.1307	.1311	.0257	.4546	.4560	.1117	.1313	.0814	.3442	.4048	.1098	.1320	.0856	.3353	.4031	.1038	.1293	.1575	.2658	.3311	460
2	.1413	.1354	.0362	.4515	.4327	.1229	.1357	.1176	.3266	.3607	.1204	.1360	.1238	.3168	.3577	.1193	.1349	.2289	.2470	.2792	459
3/4	.0742	.0737	.0158	.4531	.4505	.0638	.0739	.0501	.3399	.3935	.0628	.0743	.0525	.3312	.3918	.0598	.0727	.0962	.2614	.3178	766
2	.0780	.0749	.0187	.4543	.4367	.0673	.0748	.0604	.3323	.3694	.0660	.0750	.0636	.3224	.3668	.0646	.0741	.1175	.2520	.2892	462
2/2	.0466	.0457	.0115	.4489	.4399	.0402	.0461	.0363	.3278	.3762	.0394	.0463	.0379	.3188	.3746	.0382	.0460	.0690	.2491	.3004	859
10GY 8/6	.5302	.5600	.1396	.4312	.4554	.4695	.5793	.4334	.3168	.3908	.4629	.5828	.4526	.3089	.3890	.4531	.5876	.8207	.2434	.3157	863
4	.5840	.6002	.1659	.4326	.4446	.5203	.6181	.5252	.3128	.3715	.5126	.6211	.5498	.3045	.3689	.5115	.6262	1.0049	.2387	.2923	611
7/8	.3503	.4147	.0810	.4141	.4902	.3115	.4399	.2359	.3155	.4455	.3085	.4439	.2447	.3094	.4452	.2922	.4520	0.4332	.2482	.3839	635
6	.3861	.4298	.0963	.4233	.4712	.3409	.4507	.2948	.3138	.4149	.3360	.4536	.3090	.3059	.4129	.3275	.4623	.5618	.2423	.3420	616
4	.4241	.4433	.1173	.4307	.4502	.3793	.4571	.3761	.3128	.3770	.3741	.4594	.3956	.3044	.3738	.3752	.4628	.7297	.2393	.2952	614
6/10	.2321	.2845	.0459	.4125	.5058	.2037	.3032	.1284	.3207	.4772	.2015	.3060	.1331	.3145	.4777	.1866	.3125	.2327	.2550	.4271	634
8	.2597	.3049	.0558	.4186	.4914	.2273	.3224	.1594	.3206	.4548	.2243	.3250	.1649	.3141	.4550	.2095	.3309	.2898	.2524	.3986	751
6	.2604	.3000	.0618	.4185	.4822	.2301	.3168	.1861	.3139	.4322	.2270	.3190	.1948	.3064	.4306	.2191	.3260	.3522	.2442	.3633	624
4	.2738	.2969	.0773	.4225	.4582	.2461	.3101	.2438	.3076	.3876	.2431	.3120	.2561	.2997	.3846	.2436	.3172	.4704	.2362	.3076	621
5/8	.1421	.1785	.0288	.4068	.5107	.1260	.1910	.0833	.3147	.4772	.1244	.1925	.0869	.3081	.4768	.1175	.1978	.1544	.2501	.4211	881
6	.1699	.2016	.0371	.4158	.4935	.1490	.2141	.1081	.3161	.4544	.1469	.2158	.1129	.3089	.4538	.1392	.2212	.2017	.2476	.3935	628

4	.1796	.1943	.0476	.4260	.4610	.1595	.2025	.1487	.3123	.3966	.1573	.2037	.1560	.3042	.3941	.1554	.2069	.2853	.2400	.3195	626
4/6	.1074	.1238	.0271	.4109	.4852	.0954	.1355	.0803	.3066	.4354	.0941	.1365	.0837	.2994	.4343	.0911	.1408	.1500	.2385	.3688	727
4	.1128	.1264	.0323	.4154	.4656	.1016	.1337	.1002	.3028	.3985	.1003	.1345	.1050	.2951	.3959	.1003	.1382	.1917	.2331	.3213	630
3/4	.0612	.0709	.0161	.4127	.4786	.0544	.0756	.0483	.3050	.4240	.0536	.0761	.0504	.2975	.4227	.0523	.0785	.0905	.2362	.3546	633
2/2	.0411	.0425	.0119	.4307	.4447	.0364	.0440	.0375	.3087	.3731	.0357	.0442	.0393	.2998	.3707	.0358	.0450	.0717	.2347	.2949	937
G 8/6	.4751	.5243	.1810	.4025	.4442	.4479	.5595	.5677	.2844	.3552	.4443	.5635	.5906	.2780	.3526	.4632	.5826	1.0682	.2191	.2756	214
4	.5440	.5626	.1912	.4192	.4335	.5003	.5881	.6123	.2942	.3458	.4939	.5909	.6409	.2862	.3424	.5139	.6052	1.1743	.2241	.2639	753
2	.5584	.5537	.1868	.4299	.4263	.5075	.5694	.6053	.3017	.3385	.5001	.5714	.6345	.2931	.3349	.5177	.5789	1.1675	.2287	.2557	212
7/6	.3433	.4019	.1402	.3878	.4539	.3286	.4381	.4328	.2739	.3652	.3264	.4418	.4496	.2681	.3628	.3431	.4639	0.8087	.2124	.2871	755
4	.3703	.4072	.1403	.4035	.4436	.3491	.4340	.4431	.2847	.3540	.3462	.4370	.4621	.2780	.3509	.3624	.4518	.8400	.2191	.2731	184
2	.4095	.4095	.1424	.4260	.4259	.3755	.4231	.4626	.2977	.3355	.3705	.4248	.4853	.2894	.3317	.3870	.4317	.8945	.2259	.2520	182
6/6	.2084	.2707	.0925	.3645	.4736	.2043	.3040	.2765	.2604	.3873	.2037	.3070	.2858	.2557	.3855	.2146	.3283	.5070	.2044	.3127	206
4	.2587	.2929	.0997	.3972	.4497	.2453	.3151	.3128	.2809	.3609	.2436	.3175	.3262	.2745	.3578	.2554	.3301	.5917	.2169	.2804	204
2	.2834	.2909	.1001	.4203	.4314	.2614	.3032	.3237	.2943	.3413	.2584	.3046	.3398	.2862	.3374	.2707	.3112	.6267	.2240	.2575	202
5/8	.1158	.1631	.0528	.3492	.4916	.1134	.1875	.1521	.2503	.4139	.1128	.1896	.1568	.2457	.4128	.1182	.2067	.2747	.1972	.3447	1260
6	.1298	.1666	.0541	.3703	.4755	.1244	.1869	.1626	.2626	.3943	.1235	.1886	.1694	.2565	.3917	.1304	.2029	.3044	.2045	.3182	756
4	.1450	.1708	.0551	.3910	.4605	.1376	.1856	.1717	.2781	.3749	.1367	.1871	.1796	.2715	.3717	.1435	.1960	.3267	.2153	.2942	174
2	.1749	.1807	.0623	.4184	.4324	.1618	.1888	.2017	.2929	.3419	.1600	.1898	.2119	.2849	.3378	.1681	.1942	.3913	.2230	.2577	172
4/4	.0958	.1170	.0383	.3816	.4660	.0914	.1291	.1183	.2698	.3810	.0908	.1302	.1240	.2633	.3774	.0964	.1383	.2261	.2092	.3002	194
2	.1154	.1218	.0417	.4138	.4368	.1073	.1282	.1343	.2902	.3467	.1062	.1289	.1411	.2823	.3426	.1118	.1325	.2604	.2215	.2625	192
3/4	.0581	.0722	.0226	.3799	.4723	.0545	.0803	.0682	.2685	.3956	.0539	.0810	.0712	.2615	.3931	.0563	.0869	.1281	.2077	.3202	1264
2	.0608	.0661	.0222	.4080	.4432	.0564	.0703	.0706	.2859	.3565	.0557	.0707	.0741	.2779	.3527	.0586	.0735	.1361	.2184	.2741	188
2/2	.0329	.0348	.0126	.4097	.4332	.0300	.0373	.0396	.2805	.3487	.0293	.0374	.0414	.2714	.3457	.0310	.0395	.0753	.2123	.2711	752
10G 8/2	.5883	.5799	.1993	.4302	.4241	.5340	.5965	.6447	.3008	.3360	.5257	.5984	.6754	.2921	.3326	.5448	.6075	1.2407	.2277	.2539	938
7/4	.4088	.4389	.1629	.4045	.4344	.3864	.4672	.5197	.2813	.3402	.3825	.4699	.5429	.2741	.3368	.4067	.4879	0.9906	.2157	.2588	627
6/6	.2669	.3193	.1259	.3748	.4483	.2623	.3538	.3901	.2607	.3517	.2608	.3568	.4051	.2551	.3488	.2825	.3805	.7290	.2029	.2733	757
4	.2810	.3179	.1237	.3889	.4400	.2720	.3458	.3935	.2690	.3420	.2700	.3480	.4118	.2622	.3379	.2938	.3674	.7534	.2077	.2597	625
5/6	.1285	.1738	.0743	.3413	.4615	.1315	.2013	.2210	.2375	.3635	.1311	.2034	.2277	.2331	.3619	.1446	.2239	.4017	.1878	.2907	644
4	.1597	.1864	.0727	.3814	.4450	.1563	.2047	.2302	.2644	.3463	.1555	.2063	.2409	.2580	.3423	.1701	.2191	.4405	.2050	.2641	643
4/4	.0941	.1149	.0457	.3696	.4510	.0933	.1283	.1429	.2560	.3520	.0929	.1294	.1493	.2500	.3483	.1025	.1392	.2715	.1997	.2713	646
3/4	.0521	.0660	.0275	.3577	.4533	.0516	.0754	.0838	.2447	.3577	.0512	.0761	.0873	.2386	.3547	.0569	.0837	.1572	.1909	.2811	649
2/2	.0326	.0369	.0148	.3864	.4378	.0311	.0407	.0463	.2632	.3445	.0307	.0409	.0483	.2557	.3411	.0334	.0439	.0879	.2021	.2659	939
BG 8/2	.5955	.5887	.2146	.4257	.4208	.5473	.6082	.6981	.2953	.3281	.5394	.6101	.7318	.2867	.3243	.5673	.6219	1.3470	.2237	.2452	1107
7/4	.4169	.4367	.1736	.4058	.4251	.3963	.4634	.5613	.2789	.3261	.3924	.4657	.5880	.2713	.3221	.4240	.4839	1.0803	.2133	.2434	454
2	.4559	.4549	.1716	.4212	.4203	.4224	.4729	.5575	.2908	.3255	.4167	.4746	.5841	.2824	.3217	.4416	.4863	1.0742	.2206	.2429	768
6/6	.2738	.3143	.1406	.3757	.4313	.2731	.3473	.4458	.2561	.3257	.2716	.3498	.4650	.2500	.3220	.3034	.3743	0.8460	.1991	.2456	910
4	.2868	.3156	.1344	.3893	.4284	.2801	.3425	.4304	.2660	.3253	.2781	.3447	.4501	.2592	.3213	.3066	.3640	.8238	.2052	.2436	457
2	.3223	.3267	.1280	.4148	.4205	.3015	.3429	.4150	.2846	.3237	.2976	.3443	.4350	.2764	.3197	.3190	.3558	.8004	.2163	.2412	769
5/6	.1397	.1803	.0915	.3396	.4382	.1472	.2097	.2837	.2298	.3273	.1471	.2117	.2955	.2248	.3236	.1719	.2354	.5351	.1824	.2498	1115
4	.1667	.1908	.0874	.3747	.4289	.1667	.2112	.2773	.2545	.3224	.1659	.2128	.2893	.2483	.3185	.1862	.2281	.5270	.1978	.2423	476
2	.1875	.1958	.0781	.4064	.4243	.1782	.2077	.2538	.2785	.3247	.1764	.2087	.2664	.2707	.3204	.1915	.2171	.4918	.2127	.2411	767
4/6	.0868	.1147	.0634	.3278	.4328	.0934	.1354	.1937	.2211	.3205	.0933	.1368	.2003	.2168	.3179	.1098	.1538	.3574	.1769	.2476	478
4	.1087	.1267	.0591	.3690	.4303	.1100	.1413	.1876	.2506	.3218	.1096	.1423	.1958	.2447	.3179	.1240	.1533	.3566	.1957	.2418	480
2	.1221	.1291	.0539	.4002	.4230	.1171	.1382	.1742	.2727	.3217	.1160	.1389	.1826	.2652	.3175	.1270	.1456	.3358	.2088	.2393	770
3/6	.0482	.0640	.0380	.3209	.4262	.0525	.0767	.1165	.2135	.3123	.0524	.0775	.1210	.2088	.3090	.0635	.0886	.2179	.1716	.2394	773
4	.0515	.0624	.0321	.3528	.4273	.0528	.0716	.0997	.2357	.3196	.0525	.0722	.1037	.2299	.3161	.0607	.0800	.1876	.1850	.2437	486
2	.0602	.0665	.0298	.3848	.4249	.0591	.0727	.0953	.2602	.3202	.0586	.0731	.0998	.2532	.3158	.0656	.0780	.1831	.2008	.2388	771
2/4	.0296	.0344	.0180	.3610	.4198	.0293	.0395	.0550	.2367	.3190	.0289	.0398	.0572	.2299	.3159	.0331	.0444	.1030	.1834	.2462	488
2	.0323	.0347	.0155	.3912	.4204	.0308	.0379	.0492	.2613	.3216	.0304	.0381	.0514	.2533	.3178	.0336	.0410	.0939	.1996	.2432	772

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*—Continued

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
10BG 8/2	.6173	.6057	.2293	.4251	.4171	.5695	.6258	.7475	.2931	.3221	.5614	.6277	.7834	.2846	.3182	.5940	.6406	1.4423	.2219	.2393	940
7/4	.4052	.4256	.1892	.3972	.4173	.3953	.4555	.6179	.2692	.3102	.3924	.4579	.6481	.2619	.3056	.4366	.4795	1.1954	.2068	.2271	645
6/6	.2518	.2983	.1589	.3552	.4208	.2653	.3375	.5073	.2390	.3041	.2651	.3402	.5289	.2337	.3000	.3102	.3706	0.9631	.1887	.2254	648
4	.2795	.3045	.1431	.3844	.4188	.2792	.3316	.4650	.2595	.3083	.2778	.3337	.4873	.2528	.3037	.3145	.3538	.8973	.2009	.2260	631
5/6	.1453	.1832	.1174	.3260	.4108	.1627	.2160	.3699	.2173	.2886	.1631	.2180	.3846	.2130	.2847	.2004	.2454	.6967	.1754	.2148	658
4	.1644	.1853	.0957	.3692	.4160	.1691	.2062	.3096	.2469	.3010	.1688	.2076	.3246	.2407	.2962	.1965	.2241	.5977	.1930	.2201	651
4/6	.0891	.1135	.0730	.3233	.4118	.0992	.1349	.2276	.2149	.2922	.0993	.1361	.2361	.2105	.2887	.1216	.1543	.4256	.1733	.2199	662
4	.0991	.1146	.0603	.3617	.4183	.1027	.1290	.1933	.2416	.3036	.1025	.1300	.2023	.2357	.2991	.1200	.1417	.3715	.1895	.2238	660
3/6	.0427	.0584	.0446	.2930	.4006	.0503	.0727	.1365	.1940	.2801	.0505	.0735	.1412	.1903	.2773	.0648	.0864	.2527	.1604	.2139	668
4	.0553	.0669	.0405	.3399	.4111	.0595	.0778	.1276	.2246	.2937	.0594	.0784	.1329	.2195	.2897	.0717	.0878	.2414	.1788	.2190	665
2/2	.0355	.0403	.0211	.3667	.4159	.0357	.0455	.0663	.2421	.3086	.0354	.0459	.0692	.2353	.3047	.0409	.0505	.1261	.1880	.2321	941
B 8/4	.5768	.5749	.2457	.4128	.4114	.5481	.6025	.8080	.2798	.3076	.5419	.6046	.8484	.2716	.3031	.5934	.6252	1.5691	.2129	.2243	464
2	.6151	.5973	.2424	.4228	.4106	.5735	.6179	.7989	.2882	.3105	.5657	.6195	.8393	.2794	.3060	.6100	.6346	1.5540	.2180	.2268	797
7/6	.3851	.4085	.2067	.3850	.4084	.3877	.4436	.6773	.2570	.2941	.3857	.4460	.7103	.2502	.2892	.4433	.4737	1.3110	.1990	.2126	911
4	.4159	.4283	.2011	.3979	.4097	.4077	.4575	.6605	.2672	.2998	.4045	.4595	.6932	.2598	.2951	.4548	.4819	1.2809	.2051	.2173	467
2	.4654	.4546	.1863	.4207	.4109	.4354	.4718	.6134	.2864	.3103	.4297	.4731	.6444	.2777	.3058	.4647	.4858	1.1930	.2168	.2286	775
6/6	.2370	.2739	.1726	.3467	.4008	.2595	.3128	.5595	.2293	.2764	.2600	.3152	.5849	.2241	.2717	.3177	.3473	1.0730	.1828	.1998	469
4	.2811	.2981	.1575	.3816	.4046	.2861	.3250	.5174	.2535	.2880	.2849	.3268	.5428	.2468	.2831	.3310	.3483	1.0028	.1968	.2071	470
2	.3096	.3074	.1401	.4089	.4060	.2980	.3234	.4637	.2747	.2981	.2951	.3245	.4876	.2665	.2931	.3287	.3371	0.9048	.2093	.2147	798
5/6	.1331	.1632	.1212	.3187	.3910	.1553	.1940	.3881	.2106	.2631	.1562	.1957	.4046	.2065	.2587	.1996	.2222	.7385	.1720	.1915	1276
4	.1539	.1737	.1057	.3552	.4008	.1662	.1959	.3455	.2349	.2768	.1664	.1972	.3617	.2294	.2719	.2015	.2153	.6658	.1862	.1989	114
2	.1793	.1807	.0858	.4023	.4053	.1749	.1917	.2831	.2692	.2951	.1733	.1925	.2971	.2615	.2904	.1947	.2012	.5493	.2060	.2129	112
4/8	.0760	.0949	.0865	.2953	.3688	.0946	.1172	.2747	.1945	.2409	.0954	.1183	.2855	.1911	.2370	.1276	.1383	.5183	.1627	.1764	1273
8	.0735	.0906	.0823	.2983	.3677	.0900	.1119	.2610	.1945	.2418	.0907	.1131	.2710	.1910	.2382	.1206	.1328	.4908	.1620	.1785	2052
6	.0868	.1067	.0877	.3087	.3795	.1045	.1287	.2806	.2033	.2506	.1052	.1299	.2921	.1995	.2464	.1375	.1493	.5322	.1679	.1823	776
4	.1163	.1278	.0805	.3583	.3937	.1237	.1447	.2621	.2331	.2728	.1236	.1457	.2749	.2271	.2677	.1505	.1606	.5076	.1838	.1962	914
2	.1250	.1282	.0624	.3961	.4061	.1221	.1374	.2057	.2640	.2947	.1221	.1380	.2162	.2584	.2897	.1388	.1455	.4007	.2026	.2124	777
3/6	.0509	.0644	.0586	.2927	.3704	.0626	.0802	.1830	.1920	.2463	.0628	.0809	.1894	.1886	.2430	.0833	.0954	.3411	.1603	.1835	494
4	.0510	.0599	.0425	.3326	.3904	.0574	.0700	.1363	.2177	.2656	.0576	.0706	.1422	.2129	.2612	.0722	.0795	.2600	.1755	.1931	495
2	.0627	.0658	.0346	.3844	.4036	.0635	.0715	.1140	.2551	.2872	.0633	.0719	.1199	.2481	.2819	.0736	.0765	.2227	.1975	.2053	496
2/2	.0335	.0349	.0180	.3877	.4038	.0330	.0381	.0583	.2552	.2946	.0326	.0383	.0611	.2474	.2900	.0374	.0412	.1124	.1960	.2157	779
10B 8/2	.6610	.6302	.2745	.4222	.4025	.6208	.6514	.9106	.2844	.2984	.6122	.6526	.9575	.2755	.2937	.6699	.6706	1.7770	.2149	.2151	942
7/6	.4053	.4205	.2348	.3821	.3965	.4150	.4576	.7761	.2517	.2776	.4133	.4599	.8145	.2449	.2725	.4851	.4906	1.5072	.1954	.1976	673
4	.4285	.4267	.2092	.4026	.4009	.4186	.4526	.6921	.2678	.2895	.4148	.4542	.7271	.2599	.2846	.4693	.4757	1.3474	.2047	.2075	780
6/6	.2806	.2995	.1824	.3880	.3928	.2969	.3320	.6015	.2413	.2699	.2965	.3339	.6310	.2351	.2647	.3571	.3612	1.1668	.1894	.1916	675
4	.3127	.3173	.1699	.3909	.3967	.3140	.3415	.5626	.2578	.2804	.3120	.3430	.5910	.2504	.2753	.3618	.3635	1.0954	.1987	.1996	781
5/6	.1714	.1877	.1316	.3493	.3825	.1907	.2132	.4344	.2275	.2543	.1913	.2146	.4555	.2221	.2491	.2397	.2366	0.8423	.1818	.1794	670
4	.1894	.1959	.1131	.3800	.3930	.1963	.2134	.3768	.2495	.2714	.1957	.2145	.3964	.2427	.2659	.2329	.2292	.7371	.1942	.1911	640
4/8	.0993	.1064	.1009	.3035	.3575	.1131	.1292	.3271	.1986	.2270	.1141	.1303	.3410	.1949	.2226	.1539	.1510	.6239	.1658	.1626	669
6	.1022	.1132	.0924	.3321	.3677	.1196	.1316	.3051	.2150	.2396	.1204	.1328	.3196	.2103	.2315	.1567	.1491	.5901	.1749	.1664	782

4	.1142	.1185	.0715	.3753	.3895	.1198	.1299	.2384	.2454	.2661	.1196	.1305	.2509	.2387	.2605	.1440	.1402	.4668	.1918	.1867	652
3/8	.0506	.0630	.0750	.2684	.3340	.0697	.0812	.2382	.1790	.2088	.0704	.0820	.2467	.1765	.2054	.0997	.0990	.4454	.1547	.1537	661
6	.0547	.0632	.0598	.3081	.3555	.0673	.0765	.1929	.1998	.2273	.0677	.0771	.2009	.1959	.2230	.0907	.0804	.3669	.1659	.1635	659
4	.0581	.0623	.0450	.3511	.3768	.0642	.0708	.1480	.2268	.2502	.0642	.0712	.1551	.2209	.2451	.0805	.0789	.2863	.1807	.1770	657
2/2	.0326	.0345	.0237	.3594	.3797	.0349	.0390	.0773	.2311	.2577	.0348	.0392	.0810	.2248	.2528	.0429	.0434	.1493	.1822	.1841	943a
PB 8/2	.6675	.6223	.2709	.4277	.3987	.6218	.6380	.9034	.2875	.2949	.6122	.6386	.9509	.2781	.2900	.6686	.6535	1.7686	.2163	.2114	808
7/6	.4482	.4311	.2329	.4030	.3876	.4430	.4556	.7802	.2639	.2714	.4390	.4567	.8211	.2557	.2660	.5074	.4794	1.5282	.2017	.1906	506
4	.4800	.4551	.2232	.4144	.3929	.4614	.4743	.7466	.2743	.2819	.4559	.4751	.7860	.2655	.2767	.5138	.4930	1.4635	.2080	.1996	786
2	.5079	.4771	.2097	.4251	.3994	.4757	.4907	.6988	.2857	.2947	.4688	.4914	.7354	.2765	.2898	.5137	.5037	1.3678	.2154	.2112	807
6/8	.3097	.3055	.1926	.3834	.3782	.3214	.3310	.6461	.2475	.2549	.3200	.3322	.6798	.2403	.2494	.3860	.3558	1.2655	.1923	.1773	1307
8	.3114	.3059	.2037	.3793	.3726	.3283	.3329	.6864	.2436	.2470	.3274	.3341	.7227	.2365	.2414	.4007	.3595	1.3476	.1901	.1706	513
6	.3297	.3184	.1918	.3926	.3791	.3364	.3404	.6472	.2541	.2571	.3345	.3413	.6818	.2464	.2514	.3986	.3621	1.2727	.1960	.1781	512
4	.3608	.3426	.1779	.4094	.3888	.3520	.3589	.5973	.2691	.2744	.3483	.3596	.6293	.2605	.2689	.3987	.3751	1.1736	.2048	.1926	787
2	.3587	.3338	.1532	.4241	.3947	.3383	.3432	.5132	.2831	.2873	.3336	.3436	.5407	.2739	.2821	.3694	.3528	1.0078	.2135	.2039	784
5/10	.1867	.1969	.1681	.3384	.3569	.2188	.2263	.5620	.2173	.2247	.2202	.2277	.5899	.2122	.2194	.2887	.2546	1.0945	.1763	.1554	1303
8	.2056	.2094	.1598	.3577	.3643	.2293	.2345	.5378	.2289	.2341	.2299	.2356	.5659	.2229	.2284	.2929	.2587	1.0544	.1824	.1611	517
6	.2090	.2063	.1347	.3800	.3751	.2202	.2241	.4551	.2448	.2492	.2198	.2250	.4797	.2377	.2434	.2687	.2414	0.8965	.1910	.1717	516
4	.2133	.2051	.1143	.4005	.3851	.2135	.2170	.3859	.2615	.2658	.2121	.2176	.4071	.2534	.2600	.2484	.2286	.7617	.2005	.1845	515
2	.2137	.2013	.0973	.4171	.3929	.2052	.2087	.3277	.2767	.2814	.2029	.2090	.3457	.2678	.2759	.2287	.2159	.6464	.2096	.1979	785
4/10	.1142	.1219	.1253	.3160	.3373	.1443	.1446	.4202	.2035	.2039	.1460	.1456	.4409	.1993	.1987	.2009	.1668	.8182	.1694	.1406	532
8	.1173	.1211	.1065	.3401	.3511	.1384	.1387	.3597	.2174	.2178	.1395	.1395	.3784	.2122	.2122	.1846	.1558	.7053	.1766	.1490	522
6	.1227	.1232	.0908	.3645	.3658	.1353	.1364	.3078	.2334	.2354	.1356	.1370	.3245	.2271	.2295	.1716	.1493	.6070	.1850	.1609	521
4	.1313	.1268	.0762	.3927	.3794	.1344	.1355	.2583	.2544	.2565	.1338	.1358	.2727	.2467	.2505	.1600	.1439	.5109	.1964	.1766	520
2	.1350	.1268	.0612	.4179	.3927	.1296	.1313	.2064	.2774	.2809	.1282	.1315	.2179	.2684	.2753	.1446	.1356	.4080	.2101	.1970	519
3/12	.0574	.0627	.0907	.2725	.2974	.0832	.0802	.2983	.1803	.1737	.0847	.0808	.3109	.1778	.1697	.1246	.0977	.5700	.1573	.1233	537
12	.0563	.0608	.0872	.2756	.2975	.0814	.0775	.2909	.1810	.1723	.0830	.0781	.3043	.1783	.1679	.1233	.0946	.5622	.1580	.1213	2053
10	.0651	.0692	.0836	.2987	.3175	.0871	.0844	.2790	.1934	.1873	.0884	.0850	.2921	.1899	.1825	.1257	.0995	.5400	.1643	.1300	534
8	.0621	.0649	.0686	.3175	.3319	.0786	.0768	.2306	.2036	.1990	.0796	.0773	.2421	.1994	.1938	.1099	.0886	.4500	.1695	.1366	532
6	.0661	.0663	.0568	.3493	.3506	.0763	.0749	.1923	.2222	.2180	.0768	.0753	.2025	.2166	.2123	.1007	.0834	.3784	.1790	.1483	530
4	.0659	.0643	.0441	.3779	.3689	.0703	.0699	.1500	.2422	.2409	.0702	.0701	.1583	.2352	.2348	.0871	.0755	.2968	.1896	.1643	528
2	.0698	.0660	.0345	.4100	.3876	.0685	.0689	.1168	.2695	.2711	.0679	.0690	.1234	.2608	.2652	.0783	.0718	.2315	.2052	.1882	526
2/6	.0326	.0337	.0377	.3138	.3238	.0412	.0405	.1251	.1993	.1959	.0416	.0407	.1310	.1950	.1910	.0577	.0475	.2421	.1662	.1367	804
4	.0330	.0334	.0309	.3395	.3432	.0388	.0385	.1033	.2150	.2129	.0390	.0386	.1085	.2096	.2076	.0518	.0436	.2015	.1745	.1469	805
2	.0342	.0329	.0204	.3906	.3759	.0352	.0353	.0689	.2523	.2530	.0350	.0353	.0726	.2447	.2473	.0419	.0376	.1356	.1950	.1747	806
2	.0363	.0353	.0231	.3834	.3725	.0378	.0383	.0776	.2459	.2490	.0376	.0384	.0817	.2384	.2433	.0457	.0413	.1524	.1910	.1724	--------
10PB 8/2	.7035	.6393	.2801	.4335	.3940	.6498	.6499	.9357	.2907	.2907	.6384	.6496	.9846	.2809	.2858	.6949	.6625	1.8314	.2179	.2078	945
7/6	.4889	.4300	.2155	.4309	.3791	.4598	.4365	.7316	.2825	.2681	.4519	.4357	.7717	.2723	.2626	.5077	.4463	1.4440	.2117	.1861	796
4	.5023	.4454	.2019	.4369	.3875	.4634	.4498	.6798	.2909	.2824	.4547	.4491	.7163	.2807	.2772	.4982	.4571	1.3366	.2174	.1994	790
6/8	.3680	.3132	.1952	.4199	.3574	.3623	.3200	.6768	.2666	.2354	.3577	.3190	.7165	.2567	.2290	.4253	.3310	1.3520	.2017	.1570	800
6	.3587	.3100	.1751	.4251	.3673	.3447	.3161	.5988	.2737	.2509	.3392	.3152	.6322	.2637	.2450	.3920	.3258	1.1860	.2059	.1712	797a
4	.3849	.3354	.1681	.4332	.3776	.3606	.3394	.5708	.2838	.2671	.3541	.3385	.6023	.2735	.2614	.3974	.3467	1.1278	.2123	.1852	791
5/10	.2291	.1933	.1503	.4001	.3375	.2413	.2014	.5291	.2483	.2072	.2401	.2009	.5616	.2395	.2004	.3038	.2126	1.0662	.1920	.1344	803
8	.2419	.2054	.1464	.4075	.3460	.2476	.2126	.5110	.2550	.2189	.2455	.2121	.5415	.2457	.2123	.3029	.2230	1.0244	.1954	.1439	801
6	.2423	.2090	.1309	.4162	.3589	.2399	.2149	.4519	.2645	.2371	.2369	.2144	.4780	.2549	.2307	.2822	.2235	0.9006	.2007	.1589	798a
4	.2467	.2144	.1167	.4270	.3711	.2363	.2180	.3999	.2766	.2553	.2327	.2175	.4227	.2666	.2492	.2675	.2238	.7948	.2080	.1740	792
4/10	.1439	.1174	.1035	.3944	.3217	.1563	.1224	.3688	.2414	.1890	.1560	.1220	.3923	.2327	.1820	.2038	.1300	.7483	.1884	.1201	696
8	.1490	.1241	.0996	.3998	.3329	.1578	.1291	.3517	.2470	.2021	.1570	.1287	.3736	.2381	.1952	.2001	.1364	.7103	.1912	.1303	802
6	.1512	.1281	.0890	.4106	.3479	.1535	.1321	.3104	.2576	.2217	.1520	.1317	.3290	.2481	.2150	.1863	.1381	.6227	.1967	.1458	799
4	.1468	.1264	.0732	.4239	.3649	.1426	.1288	.2526	.2721	.2458	.1406	.1285	.2674	.2620	.2395	.1644	.1326	.5046	.2051	.1655	793
3/10	.0868	.0659	.0777	.3769	.2859	.1031	.0693	.2843	.2258	.1517	.1039	.0690	.3041	.2179	.1446	.1465	.0750	.5864	.1813	.0929	864
8	.0874	.0703	.0599	.4019	.3230	.0936	.0724	.2143	.2461	.1903	.0933	.0720	.2283	.2370	.1830	.1208	.0760	.4365	.1908	.1201	701

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*—Continued

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
10PB 6	.0855	.0696	.0528	.4114	.3348	.0883	.0711	.1874	.2546	.2050	.0876	.0708	.1993	.2449	.1979	.1100	.0741	.3801	.1950	.1313	700
4	.0841	.0700	.0469	.4184	.3483	.0841	.0713	.1643	.2630	.2232	.0831	.0711	.1743	.2530	.2163	.1009	.0739	.3308	.1996	.1461	794
2/6	.0433	.0351	.0277	.4081	.3306	.0455	.0358	.0988	.2529	.1985	.0454	.0356	.1051	.2437	.1916	.0574	.0372	.2002	.1947	.1261	707
4	.0447	.0370	.0281	.4074	.3369	.0466	.0380	.0992	.2535	.2067	.0463	.0378	.1054	.2444	.1997	.0581	.0397	.2005	.1949	.1331	878
P 8/4	.7006	.5974	.2643	.4484	.3824	.6318	.5940	.8897	.2986	.2808	.6167	.5913	.9372	.2875	.2756	.6671	.5984	1.7480	.2214	.1986	284
2	.6706	.5915	.2313	.4490	.3961	.5992	.5897	.7723	.3055	.3007	.5858	.5884	.8133	.2947	.2960	.6193	.5918	1.5143	.2272	.2171	282
7/6	.5577	.4476	.2133	.4577	.3673	.4972	.4381	.7256	.2994	.2638	.4824	.4341	.7654	.2868	.2581	.5274	.4391	1.4333	.2198	.1830	1337
4	.5314	.4492	.2013	.4496	.3801	.4790	.4457	.6795	.2986	.2778	.4674	.4434	.7162	.2873	.2725	.5072	.4487	1.3380	.2211	.1956	274
2	.4740	.4129	.1685	.4491	.3912	.4256	.4108	.5658	.3035	.2930	.4161	.4096	.5964	.2926	.2880	.4441	.4123	1.1132	.2255	.2093	272
6/8	.4211	.3232	.1848	.4533	.3479	.3846	.3157	.6363	.2877	.2362	.3735	.3118	.6721	.2751	.2297	.4246	.3187	1.2633	.2116	.1588	268
8	.4344	.3328	.1800	.4587	.3513	.3921	.3227	.6218	.2934	.2414	.3802	.3187	.6575	.2803	.2350	.4276	.3233	1.2384	.2149	.1625	2057
6	.4044	.3230	.1693	.4510	.3602	.3681	.3179	.5788	.2910	.2514	.3582	.3150	.6111	.2789	.2453	.4009	.3211	1.1465	.2146	.1718	266
4	.3984	.3330	.1562	.4489	.3751	.3610	.3301	.5296	.2958	.2704	.3522	.3282	.5588	.2843	.2648	.3863	.3329	1.0460	.2188	.1886	264
2	.3764	.3257	.1368	.4487	.3882	.3391	.3238	.4610	.3017	.2981	.3315	.3227	.4863	.2907	.2830	.3563	.3253	0.9092	.2240	.2045	262
5/10	.3004	.2129	.1422	.4583	.3248	.2768	.2039	.5008	.2820	.2078	.2680	.2002	.5310	.2683	.2003	.3156	.2049	1.0065	.2067	.1342	228
8	.2930	.2177	.1313	.4564	.3391	.2682	.2107	.4574	.2864	.2250	.2601	.2076	.4843	.2732	.2181	.2998	.2120	0.9150	.2102	.1486	226
6	.2612	.2018	.1094	.4563	.3526	.2368	.1965	.3774	.2921	.2424	.2299	.1942	.3992	.2793	.2359	.2591	.1974	.7520	.2144	.1633	224
4	.2413	.1964	.0964	.4518	.3677	.2188	.1934	.3297	.2950	.2606	.2133	.1919	.3484	.2830	.2546	.2362	.1945	.6545	.2176	.1792	222
2	.2299	.1976	.0833	.4500	.3868	.2070	.1959	.2818	.3023	.2861	.2024	.1952	.2975	.2912	.2808	.2179	.1964	.5574	.2243	.2021	220
4/12	.2177	.1362	.1054	.4740	.2966	.1966	.1252	.3748	.2823	.1797	.1885	.1213	.3976	.2665	.1715	.2256	.1243	.7550	.2042	.1125	693
12	.2321	.1474	.1060	.4844	.2999	.2101	.1336	.3802	.2902	.1845	.2007	.1292	.4041	.2733	.1759	.2372	.1307	.7721	.2081	.1146	2058
10	.2027	.1373	.0930	.4681	.3171	.1832	.1292	.3276	.2863	.2019	.1763	.1261	.3471	.2714	.1942	.2065	.1289	.6577	.2079	.1298	250
8	.1851	.1339	.0829	.4605	.3332	.1704	.1278	.2954	.2871	.2153	.1655	.1258	.3146	.2732	.2076	.1938	.1270	.6011	.2102	.1377	816
6	.1700	.1282	.0721	.4590	.3463	.1539	.1240	.2503	.2914	.2348	.1492	.1223	.2649	.2781	.2281	.1694	.1243	.5001	.2134	.1566	246
4	.1668	.1322	.0654	.4578	.3628	.1496	.1289	.2240	.2977	.2566	.1453	.1277	.2367	.2851	.2505	.1605	.1291	.4449	.2186	.1758	244
2	.1481	.1262	.0545	.4506	.3837	.1337	.1248	.1850	.3015	.2814	.1308	.1243	.1955	.2903	.2759	.1416	.1250	.3670	.2235	.1973	242
3/10	.1360	.0843	.0687	.4706	.2918	.1263	.0769	.2514	.2779	.1692	.1221	.0746	.2686	.2624	.1603	.1506	.0756	.5174	.2025	.1017	1319
8	.1189	.0806	.0579	.4620	.3133	.1105	.0759	.2072	.2807	.1929	.1070	.0743	.2204	.2664	.1850	.1284	.0756	.4207	.2055	.1210	298
6	.1079	.0777	.0502	.4576	.3295	.0999	.0744	.1778	.2837	.2113	.0970	.0732	.1889	.2701	.2039	.1141	.0744	.3593	.2083	.1358	296
4	.0966	.0730	.0414	.4579	.3460	.0881	.0705	.1446	.2906	.2325	.0856	.0696	.1533	.2774	.2257	.0977	.0704	.2900	.2132	.1538	294
2	.0909	.0749	.0341	.4546	.3746	.0817	.0736	.1166	.3006	.2706	.0797	.0731	.1232	.2887	.2650	.0869	.0735	.2313	.2218	.1877	292
2/6	.0544	.0371	.0259	.4631	.3160	.0511	.0346	.0955	.2820	.1911	.0498	.0340	.1024	.2676	.1824	.0606	.0340	.1985	.2068	.1160	306
4	.0503	.0373	.0223	.4577	.3398	.0467	.0358	.0796	.2879	.2208	.0455	.0353	.0849	.2746	.2132	.0532	.0355	.1627	.2116	.1412	1313
4	.0551	.0410	.0248	.4558	.3393	.0515	.0394	.0885	.2869	.2196	.0502	.0389	.0944	.2738	.2119	.0590	.0392	.1808	.2113	.1405	2059
2	.0460	.0363	.0184	.4569	.3599	.0422	.0351	.0650	.2966	.2466	.0412	.0348	.0691	.2843	.2397	.0465	.0347	.1316	.2186	.1632	877
2	.0503	.0390	.0208	.4564	.3544	.0460	.0378	.0733	.2930	.2406	.0450	.0374	.0779	.2804	.2335	.0511	.0376	.1483	.2157	.1587	2060
10P 8/4	.7343	.6280	.2446	.4570	.3908	.6490	.6182	.8230	.3105	.2958	.6327	.6158	.8677	.2990	.2910	.6667	.6146	1.6201	.2298	.2118	810
7/8	.6373	.4972	.1983	.4782	.3731	.5445	.4735	.6745	.3217	.2798	.5251	.4679	.7121	.3079	.2744	.5498	.4635	1.3347	.2342	.1974	1541
6	.5944	.4839	.1886	.4692	.3820	.5152	.4675	.6383	.3178	.2884	.4995	.4640	.6736	.3051	.2834	.5233	.4606	1.2697	.2331	.2052	1540
4	.5797	.4799	.1837	.4662	.3860	.5048	.4657	.6210	.3172	.2926	.4907	.4628	.6554	.3050	.2877	.5134	.4593	1.2268	.2334	.2088	1539
6/8	.4729	.3471	.1520	.4866	.3571	.4011	.3245	.5260	.3097	.2595	.3847	.3191	.5558	.3054	.2532	.4093	.3152	1.0495	.2399	.1779	1538

6/8	.4729	.3471	.1520	.4866	.3571	.4011	.3245	.5250	.3207	.2595	.3847	.3191	.5558	.3054	.2533	.4093	.3152	1.0485	.2309	.1778	1538
6	.4544	.3487	.1447	.4795	.3679	.3887	.3307	.4951	.3200	.2723	.3745	.3265	.5233	.3059	.2667	.3952	.3230	.9833	.2323	.1898	1537
4	.4220	.3372	.1332	.4729	.3779	.3637	.3238	.4515	.3193	.2843	.3518	.3209	.4764	.3062	.2792	.3683	.3179	.8916	.2334	.2015	1536
5/10	.3908	.2520	.1140	.5164	.3330	.3161	.2226	.3999	.3368	.2371	.2979	.2154	.4244	.3178	.2297	.3141	.2097	.8051	.2364	.1578	1535
8	.3559	.2406	.1052	.5072	.3429	.2905	.2174	.3644	.3331	.2492	.2751	.2116	.3858	.3153	.2425	.2886	.2075	.7286	.2356	.1694	1534
6	.3236	.2315	.0955	.4974	.3558	.2688	.2129	.3299	.3312	.2623	.2568	.2088	.3494	.3151	.2562	.2684	.2044	.6592	.2371	.1806	1533
4	.2985	.2277	.0904	.4841	.3693	.2534	.2143	.3098	.3259	.2757	.2441	.2116	.3274	.3117	.2702	.2549	.2079	.6154	.2364	.1928	1532
4/10	.2542	.1546	.0759	.5244	.3190	.2045	.1331	.2713	.3359	.2186	.1920	.1278	.2890	.3154	.2100	.2060	.1236	.5528	.2334	.1401	1531
8	.2419	.1581	.0734	.5110	.3340	.1983	.1406	.2589	.3317	.2352	.1878	.1364	.2752	.3133	.2276	.2003	.1330	.5238	.2337	.1551	1530
6	.2232	.1548	.0678	.5006	.3472	.1860	.1407	.2377	.3295	.2493	.1775	.1375	.2525	.3128	.2424	.1885	.1343	.4796	.2349	.1674	1529
4	.2031	.1525	.0627	.4855	.3646	.1730	.1428	.2170	.3247	.2680	.1668	.1408	.2299	.3102	.2620	.1759	.1381	.4342	.2350	.1846	1528
3/10	.1328	.0765	.0461	.5199	.2996	.1101	.0648	.1694	.3198	.1883	.1038	.0619	.1814	.2990	.1784	.1175	.0599	.3508	.2224	.1135	1527
8	.1331	.0810	.0431	.5175	.3149	.1095	.0700	.1560	.3265	.2087	.1034	.0674	.1666	.3065	.1996	.1139	.0652	.3205	.2279	.1306	1526
6	.1191	.0798	.0406	.4972	.3332	.1011	.0722	.1448	.3179	.2270	.0966	.0704	.1543	.3007	.2191	.1064	.0691	.2952	.2261	.1469	1525
4	.1117	.0795	.0354	.4927	.3510	.0948	.0731	.1245	.3242	.2500	.0910	.0718	.1324	.3084	.2430	.0977	.0702	.2524	.2325	.1670	1524
2/6	.0670	.0426	.0244	.4999	.3178	.0579	.0377	.0898	.3123	.2032	.0555	.0366	.0964	.2945	.1941	.0634	.0355	.1869	.2217	.1243	1523
4	.0601	.0407	.0216	.4910	.3327	.0521	.0370	.0779	.3119	.2215	.0501	.0362	.0834	.2954	.2133	.0563	.0354	.1606	.2230	.1404	723
RP 8/6	.7512	.6268	.2126	.4723	.3941	.6411	.6058	.7081	.3279	.3099	.6215	.6019	.7454	.3157	.3057	.6316	.5947	1.3862	.2418	.2276	1369
4	.7344	.6219	.2144	.4676	.3959	.6321	.6050	.7136	.3240	.3101	.6140	.6019	.7511	.3122	.3060	.6272	.5961	1.3963	.2394	.2276	1368
2	.7511	.6548	.2244	.4607	.4017	.6540	.6435	.7454	.3201	.3150	.6375	.6417	.7844	.3089	.3110	.6531	.6368	1.4572	.2377	.2318	1367
7/8	.6536	.5137	.1760	.4866	.3824	.5474	.4845	.5937	.3367	.2980	.5277	.4792	.6264	.3231	.2934	.5346	.4690	1.1712	.2458	.2157	1366
6	.6500	.5213	.1776	.4819	.3864	.5482	.4955	.5975	.3340	.3019	.5297	.4910	.6301	.3209	.2974	.5373	.4814	1.1771	.2447	.2192	1365
4	.6096	.5070	.1719	.4731	.3935	.5214	.4885	.5757	.3288	.3081	.5061	.4857	.6068	.3166	.3038	.5151	.4777	1.1319	.2424	.2248	1364
2	.5671	.4851	.1660	.4655	.3982	.4913	.4727	.5547	.3235	.3113	.4786	.4710	.5845	.3119	.3070	.4898	.4651	1.0893	.2396	.2275	1363
6/10	.5504	.3810	.1278	.5197	.3597	.4384	.3389	.4388	.3605	.2787	.4160	.3307	.4644	.3435	.2731	.4139	.3173	0.8747	.2577	.1976	1362
8	.5213	.3715	.1229	.5132	.3658	.4186	.3349	.4197	.3568	.2855	.3987	.3282	.4438	.3406	.2804	.3963	.3152	.8344	.2563	.2039	575
6	.4803	.3622	.1202	.4989	.3762	.3951	.3345	.4084	.3472	.2939	.3794	.3298	.4316	.3326	.2891	.3805	.3192	.8102	.2520	.2114	1360
4	.4552	.3641	.1247	.4822	.3857	.3850	.3454	.4214	.3343	.2998	.3726	.3424	.4450	.3212	.2952	.3789	.3347	.8333	.2449	.2164	1359
2	.3851	.3263	.1133	.4670	.3957	.3337	.3169	.3799	.3238	.3075	.3250	.3156	.4006	.3121	.3031	.3335	.3112	.7476	.2395	.2235	1358
5/10	.4308	.2724	.0840	.5472	.3460	.3283	.2301	.2934	.3854	.2702	.3073	.2219	.3116	.3655	.2639	.2980	.2074	.5915	.2717	.1890	1357
8	.3813	.2537	.0824	.5316	.3536	.2984	.2206	.2858	.3708	.2741	.2818	.2143	.3031	.3525	.2682	.2778	.2029	.5739	.2634	.1924	1356
6	.3330	.2346	.0772	.5164	.3639	.2670	.2099	.2661	.3593	.2826	.2542	.2055	.2820	.3428	.2771	.2531	.1964	.5327	.2577	.1999	1355
4	.2968	.2271	.0768	.4941	.3780	.2470	.2110	.2623	.3429	.2929	.2379	.2084	.2776	.3287	.2879	.2406	.2017	.5222	.2494	.2091	1354
2	.2692	.2225	.0762	.4740	.3918	.2304	.2138	.2559	.3291	.3054	.2237	.2125	.2698	.3169	.3010	.2280	.2089	.5035	.2425	.2221	1353
4/12	.3037	.1734	.0573	.5683	.3245	.2231	.1396	.2029	.3945	.2467	.2050	.1322	.2158	.3707	.2390	.1985	.1220	.4114	.2712	.1667	1352
10	.2776	.1650	.0547	.5582	.3318	.2072	.1359	.1934	.3862	.2534	.1916	.1297	.2059	.3634	.2461	.1871	.1206	.3932	.2670	.1721	1351
8	.2503	.1577	.0530	.5429	.3421	.1921	.1341	.1873	.3742	.2612	.1796	.1294	.1995	.3532	.2544	.1778	.1212	.3812	.2612	.1782	1350
6	.2365	.1598	.0553	.5236	.3539	.1881	.1406	.1918	.3614	.2701	.1784	.1371	.2033	.3439	.2642	.1783	.1304	.3846	.2572	.1881	554
4	.2104	.1550	.0538	.5019	.3698	.1734	.1418	.1855	.3464	.2833	.1665	.1397	.1966	.3311	.2778	.1688	.1344	.3717	.2501	.1991	1348
2	.1847	.1509	.0528	.4755	.3886	.1582	.1445	.1785	.3288	.3002	.1536	.1435	.1885	.3163	.2955	.1575	.1408	.3531	.2418	.2161	1347
3/10	.1654	.0932	.0311	.5710	.3217	.1209	.0743	.1126	.3928	.2413	.1110	.0703	.1206	.3677	.2329	.1084	.0642	.2327	.2674	.1583	1346
8	.1445	.0858	.0288	.5576	.3312	.1082	.0706	.1033	.3836	.2502	.1003	.0674	.1103	.3607	.2425	.0987	.0623	.2120	.2646	.1671	1345
6	.1210	.0776	.0257	.5396	.3460	.0935	.0664	.0906	.3732	.2652	.0877	.0643	.0965	.3529	.2586	.0868	.0603	.1845	.2619	.1818	1344
4	.1087	.0757	.0257	.5174	.3602	.0873	.0675	.0895	.3573	.2762	.0831	.0660	.0952	.3401	.2702	.0835	.0629	.1809	.2552	.1923	1370
2	.0884	.0695	.0243	.4851	.3816	.0749	.0654	.0831	.3351	.2930	.0725	.0648	.0880	.3217	.2879	.0742	.0631	.1657	.2448	.2081	1343
2/6	.0843	.0520	.0199	.5397	.3328	.0656	.0440	.0721	.3609	.2421	.0615	.0424	.0773	.3395	.2339	.0630	.0397	.1493	.2500	.1576	1342
4	.0736	.0476	.0170	.5327	.3445	.0577	.0411	.0605	.3621	.2582	.0543	.0399	.0646	.3421	.2511	.0549	.0376	.1239	.2537	.1736	1341
2	.0565	.0420	.0145	.5002	.3715	.0465	.0386	.0499	.3442	.2859	.0445	.0380	.0529	.3287	.2806	.0452	.0367	.0999	.2484	.2018	1340
10RP 8/6	.7907	.6533	.2106	.4779	.3949	.6712	.6251	.7068	.3351	.3121	.6513	.6215	.7456	.3227	.3079	.6562	.6066	1.3929	.2471	.2284	853
4	.7438	.6301	.2084	.4701	.3982	.6378	.6107	.6948	.3282	.3143	.6198	.6079	.7320	.3163	.3102	.6287	.5987	1.3630	.2427	.2311	1392
7/8	.6363	.4810	.1415	.5055	.3821	.5152	.4412	.4777	.3593	.3077	.4941	.4354	.5045	.3445	.3036	.4830	.4169	0.9452	.2618	.2259	1391

TABLE 2.—*Tristimulus specifications and trilinear coordinates of the Munsell standards for the four illuminants, A, C, "D," and "S," based on spectrophotometric data obtained at the National Bureau of Standards*—Continued

Munsell notation	For ICI illuminant *A*					For ICI illuminant *C*					For illuminant "*D*"					For illuminant "*S*"					Munsell painting number
	X	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	*X*	*Y*	*Z*	*x*	*y*	
10RP 6	.6237	.4877	.1483	.4951	.3871	.5129	.4551	.4983	.3498	.3104	.4935	.4502	.5257	.3359	.3064	.4873	.4350	.9820	.2559	.2284	1390
4	.5697	.4632	.1458	.4833	.3930	.4789	.4397	.4896	.3401	.3122	.4639	.4368	.5166	.3273	.3082	.4639	.4244	.9653	.2503	.2290	1389
6/10	.5586	.3737	.0953	.5435	.3637	.4255	.3210	.3256	.3969	.2994	.4009	.3123	.3447	.3790	.2952	.3764	.2902	.6492	.2861	.2205	1388
8	.5182	.3668	.1004	.5259	.3722	.4066	.3248	.3414	.3790	.3027	.3867	.3183	.3612	.3627	.2986	.3703	.2998	.6792	.2745	.2222	1387
6	.4751	.3553	.1039	.5085	.3803	.3835	.3240	.3521	.3620	.3057	.3676	.3195	.3721	.3471	.3016	.3589	.3049	.6983	.2635	.2238	1386
4	.4356	.3451	.1083	.4900	.3882	.3627	.3239	.3652	.3448	.3080	.3504	.3212	.3855	.3315	.3039	.3491	.3107	.7215	.2527	.2249	1385
5/10	.4431	.2690	.0594	.5743	.3486	.3208	.2180	.2048	.4315	.2931	.2971	.2088	.2173	.4108	.2887	.2691	.1888	.4111	.3097	.2173	1384
8	.4005	.2540	.0591	.5612	.3560	.2966	.2111	.2040	.4168	.2966	.2771	.2039	.2165	.3973	.2923	.2549	.1856	.4103	.2996	.2182	1383
6	.3638	.2512	.0657	.5344	.3691	.2819	.2191	.2251	.3882	.3018	.2673	.2143	.2386	.3712	.2976	.2536	.1996	.4504	.2806	.2210	1382
4	.3229	.2463	.0717	.5038	.3843	.2634	.2260	.2435	.3594	.3083	.2537	.2235	.2576	.3452	.3042	.2484	.2130	.4843	.2627	.2252	1381
4/10	.2988	.1724	.0384	.5864	.3383	.2119	.1358	.1345	.4395	.2817	.1943	.1289	.1431	.4167	.2764	.1754	.1151	.2728	.3114	.2044	1380
8	.2649	.1625	.0402	.5665	.3475	.1948	.1332	.1400	.4163	.2846	.1810	.1278	.1488	.3955	.2793	.1678	.1164	.2829	.2959	.2053	1379
6	.2310	.1495	.0379	.5521	.3574	.1740	.1262	.1312	.4035	.2924	.1633	.1222	.1393	.3844	.2876	.1532	.1125	.2643	.2890	.2123	1378
4	.1947	.1420	.0412	.5153	.3758	.1561	.1277	.1409	.3676	.3006	.1495	.1257	.1493	.3521	.2962	.1457	.1189	.2814	.2668	.2178	1377
3/10	.1756	.0958	.0237	.5952	.3246	.1223	.0735	.0849	.4356	.2621	.1107	.0690	.0907	.4094	.2552	.1010	.0612	.1744	.3002	.1818	1376
8	.1502	.0875	.0210	.5805	.3383	.1074	.0698	.0742	.4273	.2776	.0986	.0664	.0791	.4039	.2719	.0904	.0597	.1513	.3000	.1979	1375
6	.1371	.0856	.0219	.5603	.3500	.1016	.0712	.0765	.4075	.2855	.0945	.0685	.0814	.3867	.2803	.0885	.0628	.1550	.2839	.2051	1374
4	.1057	.0743	.0215	.5247	.3686	.0834	.0658	.0741	.3735	.2945	.0793	.0644	.0787	.3566	.2896	.0771	.0607	.1489	.2689	.2118	1373
2/6	.0844	.0506	.0152	.5620	.3367	.0626	.0416	.0544	.3949	.2621	.0579	.0397	.0581	.3718	.2551	.0558	.0364	.1115	.2740	.1788	1372
4	.0676	.0453	.0140	.5326	.3571	.0526	.0395	.0491	.3726	.2799	.0496	.0385	.0522	.3536	.2744	.0486	.0361	.0991	.2643	.1966	1371
N 9.6/	.9974	.9085	.3158	.4489	4089	8863	.9076	1.0461	.3121	.3196	.8684	.9075	1.1006	.3019	.3155	8983	.9066	2.0429	.2335	.2356	60
No. 57	.9166	.8344	.2867	.4498	.4095	.8123	.8329	0.9476	.3133	.3212	.7955	.8328	0.9966	.3031	.3173	.8201	.8313	1.8482	.2343	.2376	1486
N 9.4/	.9171	.8333	.2859	.4504	.4092	.8119	.8311	.9455	.3137	.3211	.7950	.8310	9944	.3034	.3171	.8193	.8292	1.8445	.2346	.2374	1486
N 9/	.7994	.7266	.2529	.4494	.4085	.7095	.7256	.8367	.3123	.3194	.6948	.7254	.8801	.3021	.3153	.7183	.7247	1.6330	.2335	.2356	1177
N 8/	.6315	.5759	.1966	.4498	.4102	.5591	.5751	.6484	.3136	.3226	.5474	.5750	.6815	.3035	.3188	.5632	.5741	1.2625	.2347	.2392	1176
N 7/	.4839	.4424	.1590	.4459	.4077	.4337	.4433	.5285	.3086	.3154	.4257	.4434	.5565	.2986	.3111	.4444	.4438	1.0349	.2311	.2308	1175
N 6/	.3324	.3032	.1076	.4473	.4079	.2972	.3032	.3579	.3102	.3164	.2916	.3033	.3768	.3001	.3121	.3036	.3030	0.7010	.2322	.2317	1071
N 5/	.2085	.1904	.0687	.4459	.4072	.1872	.1906	.2292	.3084	.3140	.1838	.1907	.2415	.2984	.3095	.1924	.1907	.4501	.2309	.2289	997
N 4/	.1323	.1207	.0438	.4457	.4068	.1189	.1209	.1462	.3080	.3131	.1167	.1209	.1542	.2979	.3086	.1224	.1209	.2875	.2306	.2278	1070
N 3/	.0717	.0651	.0237	.4469	.4054	.0643	.0651	.0792	.3084	.3119	.0631	.0651	.0835	.2981	.3073	.0662	.0651	.1558	.2306	.2267	1171
N 2/	.0333	.0301	.0109	.4479	.4051	.0298	.0300	.0366	.3091	.3114	.0292	.0300	.0386	.2987	.3069	.0306	.0300	.0720	.2310	.2262	1068
N 1/	.0168	.0153	.0056	.4461	.4066	.0151	.0154	.0185	.3075	.3137	.0147	.0153	.0195	.2970	.3093	.0155	.0154	.0364	.2298	.2291	1169

Graphs similar to figures 2 to 8 could of course be plotted for the other illuminants, using the data given in table 2. While this has not been done for the present paper, it has seemed of interest to show the effect of the illuminant on the location and shape of the network. This is done in figure 9, where the values of x and y for Munsell value 5/ are plotted to the same scale for the four illuminants.

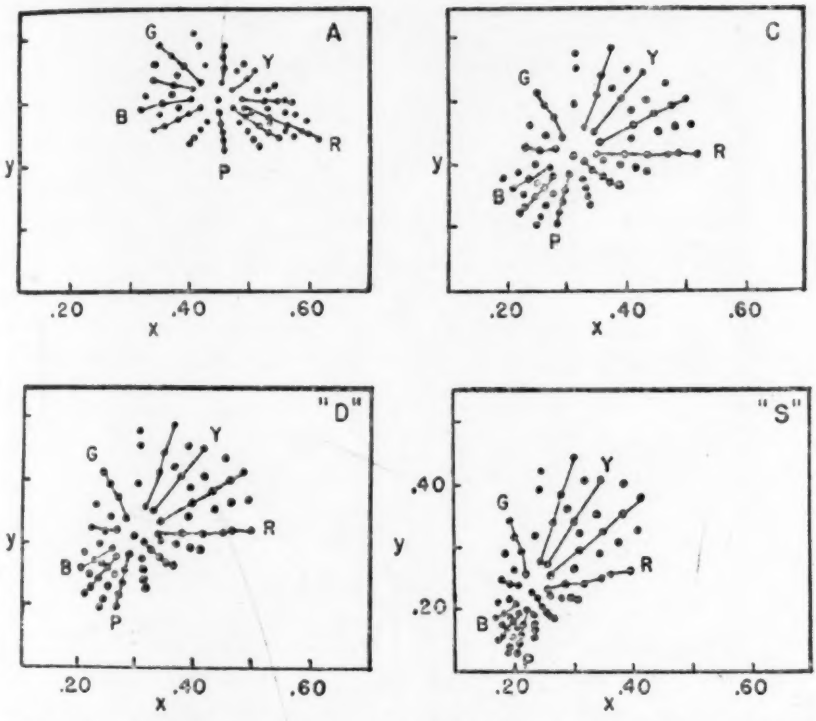

FIGURE 9.—*Values of x and y for samples of Munsell value 5/ for illuminants A, C, "D", and "S."*

This graph shows the effect of illuminant on the location and shape of the Munsell network.

V. COMPARISON WITH GLENN-KILLIAN DATA

Differences between the methods used by Glenn and Killian and those used at the National Bureau of Standards are understood to be as follows: (1) The Glenn-Killian spectrophotometric data were obtained with samples backed by "a standard white substance," the National Bureau of Standards data with samples backed with black paper, (2) the calibration curves (see above) run on each sheet at the National Bureau of Standards were not used by Glenn and Killian, and (3) the Glenn-Killian colorimetric computations were made by the selected-ordinate method, the NBS data by the weighted-ordinate method.

Spectrophotometric differences caused by the backing are illustrated in figure 10, in which are shown the curves obtained on four Munsell samples, each sample being run first with white backing and

then with black backing. The spectral reflections of the backings used for figure 10 are shown in the figure. It will be noted that the effect of backing becomes appreciable at wavelengths greater than 550 mμ approximately, if the values of apparent reflectance are greater than 0.60 or 0.65. (The slight separation of the curves for *PBP* 8/2

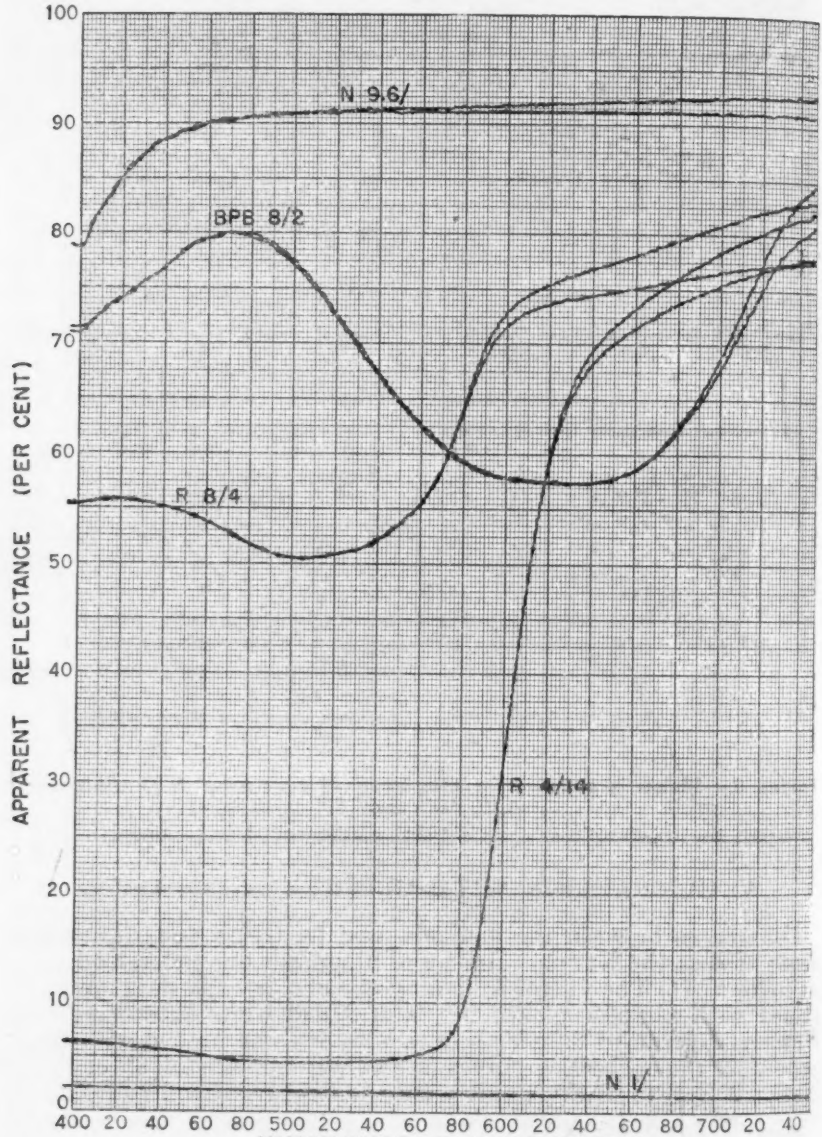

FIGURE 10.—*Effect of backing on the spectral apparent reflectance of Munsell samples.*

The upper curve of each pair was obtained with the sample backed with a white paper (*N* 9.6/), the lower curve with the sample backed with a black paper (*N* 1/). Note that no difference in curves caused by difference in backing is apparent for values of reflectance less than 0.6 or at the shorter wavelengths.

between 480 and 600 mμ is not considered significant. It is probably caused by nonuniformity of the sample. Differences of this magnitude can be obtained when a sample is rerun with the same backing if the sample and backing have been removed and reinserted between runs.)

The effects of such spectrophotometric differences on the computed values of Y, x, and y are shown in table 3. Since these samples probably illustrate the maximum effects to be expected from the two backings, it is apparent that the differences in color caused by measurement with white or with black backing are mostly unimportant.

The use of calibration curves on each record sheet—those enabling corrections of wavelength errors, 100-percent and zero curve deviations, and aging of the MgO comparison surface, as used at the National Bureau of Standards—enables spectrophotometric data to be obtained with much less care and worry regarding certain details of operation than if these calibration curves were omitted. Omission of the curves makes it necessary for the operator to take great care, for example, in the insertion of the graph sheet in the instrument, in continually checking the wavelength calibration of the instrument, and in controlling or watching the graph paper for expansion or shrinkage with change of humidity. A new MgO comparison surface must be prepared each day, and the question of reproducibility of such surfaces thus enters. The possibility of erratic differences in results between the two investigations is thus present, but since different actual samples were measured, no further conclusions can be reached regarding the erratic differences between the Glenn-Killian and the National Bureau of Standards data.

TABLE 3.—*Effect of backing on colors of Munsell samples*

[Values are computed from the spectrophotometric curves shown in figure 10.]

Munsell sample	Values obtained with white backing minus values obtained with black backing		
	ΔY	Δx	Δy
R 4/14	+0.0007	+0.0010	−0.0002
R 8/4	+.0043	+.0023	+.0001
BPB 8/2	+.0000$_1$	+.0000$_1$	.0000$_0$
N 9.6/	+.0034	+.0007	+.0003

With respect to the differences between values of X, Y, Z, x, y, and z resulting from differences in computational procedure—30 selected ordinates, as against weighted ordinates at every 10$m\mu$—it has been shown [21] that such differences are small for samples such as those considered here, much less than some of the differences shown. Only small and unimportant errors are therefore to be expected from this difference in computational procedure.

Detailed comparison of the values of x and y obtained by Glenn and Killian with those obtained at the National Bureau of Standards may be made by the inspection of figures 2 to 8 or by study of the published data. Only two additional points will be noted here.

1. Certain consistent differences in the respective chromaticities are apparent when the (x, y)-data for certain groups of samples having the

same hue designations (figs. 2 to 8) are replotted in a single graph, regardless of value level. This is particularly noticeable for the *10 GY*, *GY*, *P*, *10 RP*, and *R* samples. However, although the maximum (x, y)-difference[4] between the Glenn-Killian and the National Bureau of Standards data is $\Delta x=0.0143$ and $\Delta y=0.0156$, inspection of figures 2 to 8 shows that in the great majority of cases there is good agreement between the two sets of data. Further effort to resolve the differences would seem unwarranted.

2. Differences in the average values of Y obtained in the two investigations are shown in table 4. The greatest differences are at the extremes. That for Munsell value 8/ may be caused partially by the differences in backing. That for Munsell value 2/ may indicate a real instrumental difference relating to the zero readings of the respective instruments; none of the 33 individual differences going into this average is negative. While the individual differences on which the values of table 4 are based reached a maximum of 0.036 (sample *P 7/2*), the final average value of +0.002 for all of the data is very small.[5]

TABLE 4.

Munsell value	Average differences in Y, Glenn-Killian values minus National Bureau of Standards values
8	+0.0031
7	+.0019
6	−.0006
5	+.0018
4	+.0020
3	+.0029
2	+.0039
Average	+0.002

VI. DERIVATION OF ISCC-NBS COLOR NAMES FROM ICI TRISTIMULUS DATA

The Munsell notations for chroma and hue may be determined from figures 2 to 8 for any color whose chromaticity falls within these diagrams by plotting its trilinear coordinates on the appropriate value-level diagrams and estimating the relative position of this point with respect to the points representing the nearest samples of constant hue and the nearest lines of constant chroma. The Munsell value of the color is found by interpolation or extrapolation between the values of apparent reflectance (Y) of the Munsell standards for illuminant *C* in table 2. By referring to the color-name charts in RP1239, the ISCC–NBS color name descriptive of that color will be found. Likewise, in disk colorimetry [21], given percentages of a certain set of disks may be transformed into trilinear coordinates, plotted in a similar manner, and the corresponding color name found. Thus the ISCC-

[4] For *YR 2/2*. As is to be expected, the discrepancies in chromaticity are greatest at the lowest value level.

[5] Differences in Munsell value corresponding to the average differences in Y shown in table 4 are significant only at the lowest values. The difference, $\Delta Y=0.0039$, corresponds to $\Delta V=0.15$ at value level 2. It is believed that the NBS data are more reliable than the Glenn-Killian data at these low-value levels. For the neutral samples *N 1/*, *N 2/*, and *N 3/*, the Glenn-Killian values of Y are from 0.005 to 0.006 higher than the NBS values given in table 2. Independent check of these samples visually on the Priest-Lange reflectometer gave values lower than the Glenn-Killian values by 0.004, and closely agreeing with the NBS data of table 2.

NBS color name for a color may be found by the use of any spectrophotometer or colorimeter [22, 23] whose resultant values may be transformed into data based on the ICI standard observer and coordinate system. Likewise, any color system may be used as a comparison standard if the trilinear coordinates of each sample in that system are plotted on the (x, y)-diagrams and the ISCC-NBS color name determined through conversion to the Munsell notation.

The authors express their appreciation to Leo G. Glasser, Harry J. Keegan, and Margaret M. Balcom for running the spectral reflection curves on which the data of the present paper are based, and to Davis H. Wilson for directing the tabulating and statistical work on the Hollerith cards. They also acknowledge their indebtedness to the late Walter T. Spry, former manager of the Munsell Color Co., and to Blanche R. Bellamy, present manager of the Munsell Color Co., who have seen that nothing was lacking in samples or assistance to make this project a success.

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RESEARCH PAPER RP1550

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THERMAL EXPANSION OF SOME INDUSTRIAL COPPER ALLOYS

By Peter Hidnert and George Dickson

ABSTRACT

This paper gives data on the linear thermal expansion of some industrial copper-nickel, copper-nickel-aluminum, copper-nickel-tin, and miscellaneous copper alloys (copper-tin, copper-lead-antimony, copper-manganese-aluminum, copper-nickel-iron, copper-nickel-zinc, copper-nickel-tin-lead, copper-nickel-zinc-iron, copper-tin-zinc-lead, copper-zinc-aluminum-iron-manganese) for various temperature ranges between 20° and 900° C. The addition of 3 percent of nickel or the combined addition of 4.5 percent of nickel and 5 percent of aluminum to copper was found to have little effect on the linear thermal expansion. The effect of various treatments on these copper-nickel and copper-nickel-aluminum alloys was also small. The coefficients of expansion of two copper-nickel-tin alloys containing 20 and 29 percent of nickel were appreciably less than the coefficients of expansion of copper for temperature ranges between 20° and 600° C. Three copper alloys containing more than 28 percent of nickel showed the smallest coefficients of expansion of the miscellaneous alloys. The coefficients of expansion of the copper alloys reported in this paper were found to be between 14.9×10^{-6} and 20.4×10^{-6} per degree centigrade for the range from 20° to 100° C.

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I. INTRODUCTION

Data obtained between 1916 and 1943 on the linear thermal expansion of some industrial copper-nickel, copper-nickel-aluminum, copper-nickel-tin, and miscellaneous copper alloys, are presented in this paper. Coefficients of expansion during the heating and cooling of the samples are given for various temperature ranges between 20° and 900° C.

II. ALLOYS INVESTIGATED

The samples of copper alloys were obtained from the American Brass Co., Waterbury, Conn., American Manganese Bronze Co., Holmesburg, Philadelphia, Pa., Aterite Co., Inc., New York, N. Y., Bunting Brass & Bronze Co., Toledo, Ohio, Crane Co., Chicago, Ill., Curtis Bay Copper & Iron Works, South Baltimore, Md., National Bureau of Standards, Scovill Mfg. Co., Waterbury, Conn., Stewart Mfg. Corp., Chicago, Ill., and the U. S. Shipping Board. The length of each sample used in the determinations of linear thermal

expansion was 300 mm (11.8 in.). The cross sections of the samples, their treatments, and chemical compositions are given in table 1. Most of the values for chemical composition were furnished by the manufacturers.

III. APPARATUS

The types of precision micrometric thermal-expansion apparatus described by Souder and Hidnert [1] [1] were used for the determinations of the linear thermal expansion of the copper alloys.

IV. RESULTS AND DISCUSSION

Expansion curves on heating and on cooling were plotted from the observations made on the samples of copper-nickel, copper-nickel-aluminum, copper-nickel-tin, and miscellaneous copper alloys at various temperatures between 20° and 900° C. Coefficients of expansion and coefficients of contraction were obtained from these curves and are given in table 1. This table also shows, for most of the samples, the permanent changes in length that occurred as a result of the heating and cooling in the thermal-expansion tests.

The data on samples 1493 and 1493(a), and samples 1494 and 1494(a) indicate that the coefficients of expansion on a second heating are nearly the same as the coefficients of contraction during the previous cooling. From these data and previous experience it is believed that similar results will be obtained for most of the other copper alloys on a second heating. The coefficients obtained during the first cooling from the maximum temperature to 20° C may therefore be used for repeated heating and cooling through this temperature range. However, if an alloy is repeatedly heated to only a moderate temperature, the coefficients of expansion during the first heating should be used.

Most of the coefficients of expansion of the samples of copper-nickel alloys and copper-nickel-aluminum alloys are in close agreement with the coefficients of expansion [2] of copper reported by Hidnert [5] and Esser and Eusterbrock [6]. The average deviation of the coefficients of expansion from the corresponding coefficients of expansion of copper is $\pm 0.2 \times 10^{-6}$. These results indicate that the addition of 3 percent of nickel or the combined addition of 4.5 percent of nickel and 5 percent of aluminum to copper has little effect on the linear thermal expansion between 20° and 800° C. The linear thermal expansion is affected only slightly by the treatments of the copper-nickel and copper-nickel-aluminum alloys, even though the mechanical properties of these alloys differed significantly.

Johansson [7], Krupkowski [8], and Aoyama and Ito [9] determined coefficients of linear thermal expansion of copper-nickel alloys for several temperature ranges between −253° and +444° C. They found that the addition of nickel lowered the coefficients of expansion. For the range 18° to 444° C., Krupkowski [8] reported that the coefficient of expansion decreased slowly from 18.1×10^{-6} per degree centigrade for 0 percent of nickel to 15.1×10^{-6} per degree centigrade for 100-percent nickel. This corresponds to an average decrease of 0.03×10^{-6} in the coefficient of expansion of copper-nickel alloys for a change of 1 percent of nickel for the temperature range indicated.

[1] Figures in brackets indicate the literature references at the end of this paper.
[2] Given for reference, in last line of table 1.

TABLE 1.—*Coefficients of linear expansion of some industrial copper alloys*

Sample	Commercial name	Chemical composition									Treatment	Average coefficients of expansion a per degree centigrade									Change in length after heating and cooling a
		Cu	Ni	Al	Sn	Zn	Fe	Mn	Pb	Si		20° to 100° C	20° to 200° C	20° to 300° C	20° to 400° C	20° to 500° C	20° to 600° C	20° to 700° C	20° to 800° C	20° to 900° C	
A. COPPER-NICKEL ALLOYS																					
		%	%	%	%	%	%	%	%	%		×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	%
1489	Tempaloy 836	96.06	3.02				0.12			0.61	Cast at 1,200° C, and machined to ⅜ inch in diameter.	16.6 *17.0	17.1 *17.3	17.5 *17.7	17.8 *17.9	18.2 ------	18.7 ------	19.2 *19.0	19.5 *19.4	------ ------	+0.01
1490	do	96.06	3.02				.12			.61	Cast at 1,200° C, annealed at 850° C for 3 hours, quenched in water, heat treated at 450° C for 3 hours, cooled slowly in air, and machined to ⅜ inch in diameter.	15.6 *17.2	16.5 *17.2	17.1 *17.6	17.6 *17.9	18.0 ------	18.6 ------	19.3 *18.9	19.4 *19.1	------ ------	+.03
1396	do	96.00	3.14							.86	5/16 inch diameter rod annealed at 850° C and quenched.	16.6	16.7	17.2	17.3	17.8	18.2	18.9	19.3	------	------
1395	do	95.71	3.22							.69	Drawn from ⅜ to 5/16 inch in diameter.	16.9 ------	17.0 *17.1	17.7 *17.5	18.0 *17.8	18.3 ------	18.7 *18.3	19.1 *18.6	19.9 *18.7	------ ------	+.10

See footnotes at end of table.

TABLE 1.—*Coefficients of linear expansion of some industrial copper alloys*—Continued

Sample	Commercial name	Chemical composition									Treatment	Average coefficients of expansion a per degree centigrade									Change in length after heating and cooling b
		Cu	Ni	Al	Sn	Zn	Fe	Mn	Pb	Si		20° to 100° C	20° to 200° C	20° to 300° C	20° to 400° C	20° to 500° C	20° to 600° C	20° to 700° C	20° to 800° C	20° to 900° C	
B. COPPER-NICKEL-ALUMINUM ALLOYS																					
		%	%	%	%	%	%	%	%	%		×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	×10⁻⁶	%
1491	Tempaloy 841	89.67	4.47	5.04						0.82	Cast at 1,200° C, and machined to ⅜ inch in diameter.	16.4 *16.8	16.8 17.0	17.4	17.8 *18.0	18.3	18.8	19.1 *19.2	19.5 *19.6		−0.01
1492	do	89.67	4.47	5.04						.82	Same treatment as sample 1490.	16.7 *16.8	*17.1 *17.1	17.5 *17.6	18.1 *17.9	18.6	19.5	19.2 *19.0	19.1 *19.4		−.02
1493	do	89.67	4.47	5.04						.82	Hot rolled at 750° C from 1½ to ¾ inch in diameter, annealed at 800° C, quenched in water, drawn to ⅝ inch in diameter, annealed, drawn to ½ inch in diameter, annealed, drawn to ⅜ inch in diameter, annealed at 850° C for 3 hours and quenched.	17.0 *16.6	16.8 *17.1	17.2 *17.5	17.8 *17.9	18.3 *18.2	19.0	19.5 *18.7			+.05
1493 (a)	do	89.67	4.47	5.04						.82	Same treatment as sample 1493, then heated to 700° C and slowly cooled.	16.8 *16.5	17.0 *17.0	17.4 *17.4	17.9 *17.9	18.3	18.5	18.7	19.1 *19.4		−.03
1494	do	89.67	4.47	5.04						.82	Same treatment as sample 1493, then heat treated at 450° C for 3 hours and slowly cooled in air.	16.8 *16.6	17.0 *17.0	17.4 *17.5	17.9 *17.9	18.3	19.1	19.7 *19.0	19.8 *19.4		+.03
1494 (a)	do	89.67	4.47	5.04						.82	Same treatment as sample 1494, then heated to 800° C and slowly cooled.	16.6 *16.4	16.9 *17.1	17.3 *17.5	17.9 *17.9	18.2	18.7	19.0 *19.1	19.3 *19.5		−.01
C. COPPER-NICKEL-TIN ALLOYS																					
1187	Admic (admiralty nickel).	69.57	28.70		0.91						Rod 1 3/10 inches in diameter, extruded from 1⅜ inches in diameter and finished hard, with a 1/16-inch draught after annealing.	*15.2	*15.4	*16.3							+0.00
1085		68	20		12						Bar ¼-by ¼-inch cross section, cast.		16.2		16.9		18.1			22.9	

D. MISCELLANEOUS COPPER ALLOYS

B. MISCELLANEOUS COPPER ALLOYS

1445	Red brass	84.96			5.02	5.15			4.87		Cast rod	[d]17.8 *17.7	[d]18.1 *18.0	[d]18.5 *18.2	[d]18.9 *18.6	[d]19.3	[d]20.0 *19.4	[d]20.6 *19.8			+0.05
1575		84.93		5.5			0.22	9.4			Cold-drawn wire 1 mm in diameter, baked in air at 140° C for 18 hours.	*18.1 *17.4									+.01
1444A	Bronze	84.84			14.95				0.21		Outside section (cross section, circular segment with 5/32-inch rise) of 7/8 inch diameter, cast rod.	[d]18.0 *17.7	[d]18.2 *18.2	[d]18.6 *18.7	[d]18.8 *19.1	[d]19.3 *19.6	[d]19.8 *20.0				−.02
1084		71	18		6	1			4		Bar 1/4-by 1/4-inch cross section, cast.		16.4		17.0		18.5			24.6	
1073	Manganese bronze	66.2	(f)	4.8	(f)	*22.9	2.6	3.5	(f)		Rod 3/8 inch in diameter, machined from 1 3/8 inch diameter casting.	18.7	19.8	20.8							−.00
897[b]	Hytensl bronze	63–68		3–6	[i]0.50	(j)	2–4	2.5–5	[i]0.20		Extruded rod, 3/8 inch diameter.	18.5 *20.4	17.0 *20.6	17.0 *21.5							−.13
897 (a) [b]	do	63–68		3–6	[i].50	(j)	2.4	2.5–5	[i].20		Same treatment as sample 897, then heated to 300° C and cooled, and aged at room temperature for 21 years.	19.0 *19.0	20.6 *20.3	21.3 *21.3							−.00
683	Wrought Aterite	65	11			22	1.5	0.5			Cold-rolled and cold-drawn rod, 1/2 inch in diameter.	[k]16.5	[k]17.0	[k]17.4				19.1	19.7		
1026	Everbrite	64.9	28.5				4.9	.9			Bar 1/2 by 7/16-inch cross section, cast.			15.8	16.2			18.0			+.01
934[l]	Stewart bearing metal.	64							34		Rod 3/8 inch in diameter, machined from sand casting.	[k]20.1	[k]20.0	20.0	[k]19.9	[k]19.8					
280	Aterite	63.60	13.18			*21.45	1.72		0.05		Rod 1/2 inch in diameter, treatment not known.	17.2	17.5	18.0 *18.1							−.00
281[m]	do											17.2 *17.4	17.6 *17.8	(n)							−.00
498	Nickel silver	58.4	15.7		(f)	*25.5	0.22	.15	(f)		Tube 3/16-inch outside diameter.	17.4	*17.4	17.8	18.3 *19.1	18.9 *19.7					−.04
682	Aterite No. 9	55	32			7	6				Bar 1/2-by 1/2-inch cross section, cast.	[k]14.9	[k]15.4	[k]16.0		17.3		18.3	18.6	19.2	
621[o]	Aterite No. 4	48.80	31.26			8.93	8.83	0.16	2.23	0.66	Bar 3/8-by 3/8-inch cross section, cast.		15.4	16.1	16.6	17.1	17.6	18.2	18.9		
	Copper [5, 6][p]	100										16.8	17.3	17.7	17.9	18.3	18.6	19.0	19.4	19.8	

[a]The values indicated by asterisks were determined from the curves on cooling the samples from the maximum temperature to 20° C.

[b]Determined from the expansion curve on heating and the contraction curve (or final observation) on cooling. The plus sign indicates an increase in length and the minus sign a decrease in length.

[c]Reported by Hidnert and Sweeney [2] to the American Physical Society.

[d]Reported by Hidnert [3] to the American Physical Society.

[e]Reported on page 159 of publication by Thomas [4].

[f]Not detected.

[g]By difference.

[h]Chemical composition within range specified by Federal Specification QQB721a, Class B, or Navy Specification 46B15, Class B.

[i]Maximum.

[j]Remainder.

[k]Computed from a second-degree equation derived from the observations.

[l]Antimony 2 percent. After heating the sample to 500° C and cooling to 20° C incident to the thermal-expansion determinations, several small globules (probably lead) were found on the surface of the sample.

[m]Duplicate of sample 280.

[n]Heating coil of oil bath burned out before observation could be taken at 300° C.

[o]Carbon 0.41 percent. (Apparent total contents of elements 101.3 percent.)

[p]Added for comparison with the copper alloys.

The coefficients of expansion of the copper-nickel alloys of the present investigation are in good agreement with comparable coefficients of expansion obtained by these observers.

The coefficients of expansion of the two copper-nickel-tin alloys containing 20 and 29 percent of nickel are appreciably less than the coefficients of expansion of copper for temperature ranges between 20° and 600° C. For the range 20° to 900° C the coefficient of expansion of the copper-nickel-tin alloy containing 20 percent of nickel is about 16 percent higher than the coefficient of expansion of copper.

The substitution of 5 percent of zinc and 5 percent of lead for 10 percent of tin in the bronze containing 15 percent of tin caused slight changes in the coefficients of expansion. The three miscellaneous alloys (samples 1026, 682, and 621) containing more than 28 percent of nickel show the smallest coefficients of expansion of this group in table 1. The coefficients of expansion (or contraction) of samples 1073, 897, 897(a), and 934, which do not contain nickel, are the largest of the alloys investigated. The coefficients of the latter four samples exceed the coefficients of expansion of copper, and the coefficients of expansion of the former three alloys are less than the coefficients of expansion of copper. The coefficients of expansion of samples 683, 280, and 281 are larger than the corresponding coefficients of expansion reported by Cook [10] for annealed copper-nickel-zinc alloys.

Data on the linear thermal expansion of other copper alloys investigated at the National Bureau of Standards have been published in previous papers [5, 11, 12, 13, 14].

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OPTICAL ROTATION AND ATOMIC DIMENSION: THE FOUR OPTICALLY ACTIVE 2-HALOGENOPENTANES

By Dirk H. Brauns

ABSTRACT

In previous articles by the author it is suggested that certain optically active halogen derivatives may be divided into two classes. The first class has the halogen directly attached to an asymmetric carbon atom. For these substances the writer has formulated the rule that the differences of their specific rotations, (Cl-F), (Br-Cl) and (I-Br), have the same numerical relation as the respective differences of the radii of the covalent-bonded atoms (41:17:21). The second class of optically active halogen derivatives has the halogen attached indirectly (by a chain of atoms) to an asymmetric carbon atom. For these latter substances the rule was formulated that the differences of their molecular rotations have a numerical relation, which likewise agrees with that for the respective differences of the radii of the neutral halogen atoms.

As these rules were established for halogen derivatives of carbohydrates, which contain several asymmetric carbon atoms, it was found desirable to investigate halogen derivatives, which contain only one asymmetric carbon. The results of the last investigation (see footnote 1) showed that the derivatives of the amyl alcohol, 2-methylbutanol-1, checked the above rule for the second class very well. The present communication reports the results obtained in checking the above rule for the first class, with the halogen derivatives of the other active amyl alcohol, pentanol-2. Pure optically active *D*- and *L*-pentanol-2 were prepared and halogenated by different methods in order to obtain the 2-fluoro-, 2-chloro-, 2-bromo- and 2-iodopentanes of highest rotation.

In spite of the difficulty that a partial inversion takes place in the halogenation, resulting in a mixture of *D* and *L* halogen derivative, the conclusion can be drawn that rule I is not contradicted by the values found, since the deviations can be plausibly explained by the incompleteness of the Walden inversion. Rule II should not be applicable to this type of compound, and the results confirm this.

It is further shown that all halogen derivatives of pentanol-2 of like configuration have the same sign of optical rotation. The specific gravity, refractive index, and boiling points at various pressures were also determined.

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I. INTRODUCTION

It is suggested in the writer's previous articles on this subject [1, 2] that certain halogen derivatives may be divided into two classes. The compounds which compose the first class have the halogen directly attached to an asymmetric carbon atom and differ only in having one halogen replaced by another. For these *D* (or *L*)[3] substances the differences of their specific rotations (Cl-F), (Br-Cl), and (I-Br) have the numerical relation 41:17:21, which agrees with the numerical relation 41:17:21[4] for the differences in atomic radii of the respective neutral or nonionized or covalent halogen atoms (rule I). Those in the second class have the halogen atom attached indirectly (by a chain of atoms) to an asymmetric carbon atom. For these latter *D* or *L* substances the differences of their molecular rotations have a numerical relation, which likewise agrees with that for the differences in atomic radii of the respective covalent-bonded halogen atoms (rule II).

These two rules for the two classes of compounds were formulated by the investigation of halogen derivatives of the carbohydrates. Inasmuch as these halogen derivatives contain several asymmetric carbons, it was found desirable to prepare and test the halogen derivatives of the two active amyl alcohols, 2-methylbutanol-1 and pentanol-2, since these compounds are simple in structure and contain only one asymmetric carbon atom.

The results of an investigation (see footnote 1) of the pure active halogen derivatives of 2-methylbutanol-1, in which the halogens are indirectly attached to the asymmetric carbon, showed that the differences of the molecular rotations have a numerical relation which agrees with that for the differences in atomic radii of the respective neutral halogen atoms as required by rule II.

The present paper reports the results obtained with the active halogen derivatives of pentanol-2, for which compounds rule I should be applicable, since the halogen is directly attached to the asymmetric carbon atom.

II. EXPERIMENTAL PART OF INVESTIGATION

1. PREPARATION OF THE COMPOUNDS

(a) DEXTRO- AND LEVOROTATORY PENTANOL-2 (METHYLPROPYLCARBINOL)

Four and a half kilograms of inactive pentanol-2 (methylpropylcarbinol) was prepared from propyl bromide and acetaldehyde according to the Grignard method [5] in batches that could be worked in 2-liter flasks. The preparation was kept for a few months over

[1] J. Research NBS **18**, 315 (1937) RP978.
[2] J. Am. Chem. Soc. **47**, 1285 (1925); BS J. Research **7**, 573 (1931) RP358.
[3] *D* and *L* refer to configuration.
[4] For a discussion of the values used, see section III, p. 98.
[5] R. H. Pickard and J. Kenyon, J. Chem. Soc. **99**, 55 (1911).

dry potassium carbonate, for resinifying small amounts of aldehyde, which is hard to separate by fractionation. The preparation then gave directly on distillation a product having the correct boiling point. The yield was about 40 percent of the theoretical amount. In the course of the investigation there were prepared from the inactive alcohol 1,200 g of pure dextrorotatory and 600 g of pure levorotatory alcohol in batches of about 200 to 300 g. This large amount of active alcohol. requiring more than 2 years of fractional recrystallization of the brucine salt of the acid phthalate of methylpropylcarbinol, was needed because it was necessary to determine which method of halogenation causes the least racemization.

The remaining part of the inactive alcohol was used for certain preliminary experiments. The pure active alcohol is stable, although some of the preparations, even when stored in the refrigerator, diminished slightly in rotation (less than 0.4% in 1 year), but this change may be due to the presence of small amounts of water, which are difficult to remove.

The resolution to the active dextro- and levorotatory alcohols was performed according to the method of Pickard and Kenyon (see footnote 5), which consists of the fractional crystallization of the isomeric brucine amyl phthalates. Although the directions given by the authors were found to be reliable, the writer has made the two improvements mentioned below.

1. It is advantageous for increasing the yield and decreasing the time of reaction to use for the esterification of the alcohol pure, freshly distilled phthalic anhydride free from phthalic acid.

2. The following procedure has been found useful for evaluating the numerous fractions of the brucine amyl phthalate used for separating the *D* and *L* forms of the alcohol.

As soon as the crystals of a fraction are separated from the mother liquor, a sample of about 1 g is crushed in a dish with a small glass pestle and allowed to dry for 30 minutes in air, with occasional stirring. A solution containing exactly 0.6 g of this sample is made up to 25 ml at 20° C with chloroform, placed in a water-jacketed 4-dm tube at 20° C, and read with a polariscope. The reading varies for the fractions from −1.38 circular degrees for the pure compound of the *D* alcohol to −2.41 circular degrees for the pure compound of the *L* alcohol. This relatively small difference in rotation is nevertheless a great help in estimating the extent of separation of the fractions. For obtaining these results it is necessary to maintain uniform conditions not only as to concentration and temperature (since the rotation of the brucine salts is greatly influenced by these factors) but also as to the time used for powdering and drying the sample. The method of drying is of importance because it was found that the brucine salts crystallize with varying amounts of acetone of crystallization, and part of this is readily volatilized.

An investigation of the brucine amyl phthalates has revealed that the pure brucine salt of the acid phthalic ester of dextrorotatory methylpropylcarbinol crystallyzes with 2 molecules of acetone when a warm acetone solution is cooled. By longer standing of the mother liquor at room temperature, crystals with 1 molecule of acetone to 2 molecules of brucine salt may separate, whereas the mother liquor in the refrigerator may again yield the compound with 2 molecules of

538330—43——3

acetone. When 0.6 g of the freshly prepared compound with 2 molecules of acetone, $C_{36}H_{42}O_8N_2+2(C_3H_6O)$, is freed from acetone by drying to constant weight in a vacuum oven at 75° C and made up to 25 ml at 20° C with chloroform, it has a specific rotation of —17.5°, expressed on the acetone-free basis. The compound with 2 molecules of acetone loses 1 molecule of acetone easily in air; the other molecule is lost by standing in vacuum at room temperature for several weeks, or more quickly in a vacuum oven at 75° C.

Usually, the pure brucine salt of the acid phthalic ester of the levorotatory methylpropylcarbinol crystallizes from a warm acetone solution by cooling, with less than 2 molecules of acetone. The results of the quantitative determinations for different samples vary, and hence no definite conclusion as to the composition can be drawn. It was also found for the compound of the levorotatory alcohol that 1 molecule of acetone is strongly bound, but is lost by drying in a vacuum oven at 75° C. When 0.6 g of the pure brucine salt containing the levorotatory alcohol is freed from acetone by drying in a vacuum oven at 75° C, and made up to 25 ml at 20° C with chloroform, it has a specific rotation of —25.9°, expressed on the acetone-free basis. These results were obtained as an average of many determinations. The existence of these acetone compounds explains why the uniform procedure is necessary for the polariscopic evaluation of the acetone-containing brucine salts. Only 1 molecule of acetone of crystallization is left after powdering and drying the substance in air for 30 minutes. The method described is sufficiently accurate for the evaluations of the fractions.

The writer's results are in accord with those of Pickard and Kenyon,[6] with the exception of the rotation of the hydrogen phthalic ester of dextrorotatory methylpropylcarbinol, concerning which Pickard and Kenyon made the following remark:

> Determinations of the molecular rotatory powers of the hydrogen phthalic esters in chloroform solution gave results which run practically parallel with those of the alcohols in the pure state, although in the case of the esters the first member occupies apparently a normal place in the series and has not, as in the case of the alcohols (as also in so many series described by other investigators) an abnormal rotatory power.

If the values for the specific and the molecular rotations of the hydrogen phthalic ester of dextrorotatory methylpropylcarbinol: $[\alpha]_D=+36.94$ and $[M]_D=+87.2$ given by Pickard and Kenyon are replaced by $[\alpha]_D=+43.05$ and $[M]_D=+101.6$, which were found by the author for the concentration and in the same solvent as used by Pickard and Kenyon, and checked many times in the course of this investigation, it is readily seen (table 1) that now also for the hydrogen phthalic esters, the first member occupies an abnormal place in the series just as in the case of the alcohols.

Since the melting point of the ester agrees with that reported by Pickard and Kenyon, it seems that the difference in the optical rotations must be due to an error in the measurement of the optical rotation.

(b) DEXTROROTATORY 2-FLUOROPENTANE

In the preparation of 2-fluoropentane, the method previously used for preparing 1-fluoro-2-methylbutane (see footnote 1) by the action

[6] R. H. Pickard and J. Kenyon, J. Chem. Soc. **99**, 52 (1911).

TABLE 1.—*Comparison of rotations of a homologous series of hydrogen phthalates of active alcohols*

Hydrogen phthalate of—	Rotation in chloroform (5%). Average values			
	$[\alpha]_D$		$[M]_D$	
	Pickard and Kenyon	Brauns	Pickard and Kenyon	Brauns
Methylethylcarbinol	33.53		74.5	
Methylpropylcarbinol	36.94	[a] 43.05	87.2	101.7
Methyl *n*-butylcarbinol	43.39		108.5	
Methyl *n*-amylcarbinol	43.92		116.0	
Methyl *n*-hexylcarbinol	42.94		119.3	
Methyl *n*-heptylcarbinol	41.11		120.0	
Methyl *n*-octylcarbinol	39.06		119.5	
Methyl *n*-nonylcarbinol	37.27		119.2	
Methyl *n*-decylcarbinol	35.59		[b] 121.0	119.0
Methyl *n*-undecylcarbinol	35.34		123.1	

[a] The constants of optically active pentanol-2 and its halogen derivatives of highest purity prepared for this investigation are recorded in tables 2 and 3.
[b] Error in calculation, corrected in next column.

of silver fluoride on the corresponding bromine derivative was improved by following a modification by Bockemueller,[7] who employed finely divided silver fluoride mixed with pure calcium fluoride. A short description of the preparation of the reagent follows. Pure calcium fluoride free from silica was prepared by the addition of a hot solution of calcium chloride to pure hydrofluoric acid solution in a platinum dish; the precipitated calcium fluoride was separated by decantation, collected on a filter and washed with water until free from chloride. A concentrated silver fluoride solution was prepared by dissolving silver carbonate in pure aqueous hydrofluoric acid in a platinum dish. The calcium fluoride was added to the silver fluoride solution in the proportions of 3 to 2, and the mixture was dried in a shallow platinum dish suspended by platinum wires from the stopper of the cover of a large paraffined desiccator provided with potassium hydroxide pellets. The desiccator was evacuated at first with a water pump and after a few days with an efficient oil pump protected by a soda-lime tower.

From time to time the material was stirred and finally powdered with a platinum pestle. The preparation was made in subdued light and kept in the dark. This procedure differs from the one given by Bockemueller, in that no heat was applied for drying the material.

The following method was used for preparing the fluorine derivative: Thirty grams of 2-bromopentane having a specific rotation of −35.9 (which is 99% of that of the highest-rotating 2-bromopentane, with $[\alpha]_D^{20}=-36.2$) was mixed with 90 ml of pure dry acetonitrile. A 250-ml platinum bottle containing 75 g of the silver fluoride-calcium fluoride mixture was closed with a rubber stopper and cooled in ice water. The solution of 2-bromopentane in acetonitrile was quickly added, and the tightly closed bottle (inserted in a Thermos bottle containing cold water) was shaken for 22 days. The temperature of the water was kept at 5° to 15° C. Three portions, 3 g each, of the silver fluoride-calcium fluoride mixture were added at about equal

[7] W. Bockemueller, Liebigs Ann. Chem. **506**, 20 (1933).

intervals during the period of the shaking. Also the rotation of a small filtered sample was determined in a 0.2-dm tube at these intervals in order to determine the end of the reaction. When the end was reached, the mixture was filtered on a fritted-glass disk and the flask cooled with ice water.

After removing the filtrate, a slight suction (600-mm pressure) was applied for washing with acetonitrile. About 80 ml of solution rotating +0.65 sugar degrees (°S) in a 0.2-dm tube was thus obtained. The filtrate was then distilled in a platinum distilling apparatus. The retort was heated gradually in an oil bath to 120° C, and the condenser and receiver were cooled with ice water. The distillate was kept overnight in contact with potassium carbonate in the icebox and then filtered. The filtrate was distilled and then fractionated at ordinary pressure from a Pyrex fractionating column, which has been formerly described. (See footnote 1.) The condenser and receiver were cooled with ice water. Finally, a microfractionating column with fine platinum-ruthenium spirals was used. For preventing bumping, a slow stream of dry nitrogen was introduced through a fine capillary. This procedure was applied for the fractionation of all halogen derivatives. 2-Fluoropentane can be distilled at ordinary pressure without decomposition. The yield was about 3 g. The constants are recorded in tables 2 and 3. The quantitative fluorine determination was performed according to a new procedure.[8] Weight of substance: 0.1104 g; loss by etching: 0.0221 g; calculated for $C_5H_{11}F$: F, 21.09 percent; found: 21.1 percent.

(c) LEVOROTATORY 2-CHLOROPENTANE

Miss Sherrill[9] applied a method of low-temperature chlorination for inactive secondary heptylalcohol, which is an adaptation of the method of Norris and Taylor[10] modified by Lucas.[11]

Following this procedure, 20 g of active methylpropylcarbinol ($[\alpha]_D^{20}=+13.8$) was added to an ice-cold solution of 62 g of zinc chloride in 46 g of 36-percent hydrochloric acid in a 300-ml Erlenmeyer flask. The flask was closed with a glass stopper and shaken for 7 hours at 0° C. The oily layer was separated after standing overnight in a separatory funnel, and shaken with a new solution of 62 g of zinc chloride in 46 g of 36-percent hydrochloric acid for 7 hours at 0° C. After separating the oily layer, the process was repeated for a third time. The oily layer was washed three times with concentrated hydrochloric acid, afterward four times with ice-cold water, once with ice-cold 10-percent sodium hydroxide solution, twice with cold 15-percent sodium chloride solution, and finally washed once with cold water. The product was dried over a mixture of calcium chloride and potassium carbonate, filtered, distilled, and finally fractionated at a pressure of 250 mm. About 17 g of a chlorine derivative was obtained.

A quantitative chlorine determination by the Carius method gave the following results: Substance, 0.2260 g; AgCl, 0.3034 g. Calculated for $C_5H_{11}Cl$: Cl, 33.28. Found: Cl, 33.21 percent. The rotation of the product in a 0.2 dm-tube at 20° C was −3.0° S, which is very low compared with the rotation of the product obtained by the follow-

[8] D. H. Brauns, J. Research NBS **27**, 105 (1941) RP1406.
[9] Mary L. Sherrill, J. Am. Chem. Soc. **52**, 1985 (1930).
[10] J. F. Norris and H. B. Taylor, J. Am. Chem. Soc. **46**, 753 (1924).
[11] H. J. Lucas, J. Am. Chem. Soc. **51**, 249 (1929).

ing method of chlorination. Inasmuch as zinc chloride acts not only as a catalyst but as a dehydrating agent, it leads to the formation of amylenes, which in turn combine with hydrogen chloride with the formation of isomeric chlorine derivatives.

The second procedure for converting active pentanol-2 to its chlorine derivative by the use of dry hydrogen chloride without zinc chloride gives better results. It was found that hydrogen chloride reacts with the alcohol at a temperature of 75° to 80° C, and thus it is necessary to seal the reaction tube to prevent loss. These preliminary experiments were made with inactive methylpropylcarbinol. Subsequently, experiments with the active alcohol were made for finding the conditions under which the highest-rotating chlorine derivative is formed. The following procedure was finally adopted:

Twenty grams of active methylpropylcarbinol ($[\alpha]_D^{22} = +13.8$) was cooled in a Pyrex pressure tube with ice and salt, and saturated with a slow stream of dry hydrogen chloride. The tube was sealed and heated gradually to 80° C in a shaking machine and kept at this temperature with shaking for 2.5 hours. The tube was then cooled, opened, and the upper layer separated from the aqueous layer in a separatory funnel. The upper layer was extracted twice with 8 ml of cold 10-percent sodium hydroxide solution, and then repeatedly with ice water. The product was separated from adhering drops of water, by pouring into a dry Erlenmeyer flask and decanting therefrom, after which it was dried over potassium carbonate for several hours and then filtered. The filtrate was first distilled and then fractionated at 250-mm pressure. The rotation of the product was not changed by additional fractionation. The preparation was repeated with another 20 g of active alcohol for obtaining a sufficient amount of chlorine derivatives. The total yield was 7 g. A quantitative chlorine determination by the Carius method gave the following results: Substance, 0.2071 g; AgCl, 0.2771 g. Calculated for $C_5H_{11}Cl$: Cl, 33.28. Found: Cl, 33.10 percent. The rotation of the product in a 0.2-dm tube at 20° C was −15.2° S, which is about five times that found for the product obtained by the zinc chloride-hydrogen chloride method of chlorination.

Chlorination with thionyl chloride and with phosphorus pentachloride did not give satisfactory results, whereas the chlorination with phosphorus trichloride gave the highest-rotating product. The last method was studied as before, first with inactive methylpropylcarbinol, and finally with the active alcohol. Thus it was found that heating the alcohol with phosphorus trichloride at 95° C is necessary for obtaining a sufficient rate of reaction. The following method was finally adopted.

Twenty grams of active methylpropylcarbinol ($[\alpha]_D^{20} = +13.6$) was cooled in a Pyrex pressure tube with ice water, and 18.9 g (12 ml) of phosphorus trichloride was added drop by drop from a small burette; calcium chloride tubes were used for excluding moisture. The pressure tube was closed with a calcium chloride tube and allowed to attain room temperature. It was sealed, shaken for 2.5 hours at 95° C, then cooled in ice and salt, and opened. The chlorinated product was poured from the sirupy phosphorous acid into a beaker of cracked ice and some water, under stirring. The product was repeatedly shaken with small portions of ice water in a

separatory funnel until the water was neutral to litmus paper. The product was separated from adhering drops of water, and dried over potassium carbonate. The ice water used for the purification was extracted with ether. The ether solution was dried and fractionated, and the yield of the alcohol was thus increased to about 17 g.

Since the chlorine derivative thus obtained had a higher specific rotation than the product of any other method of chlorination, 170 g of active alcohol in 30-g batches was converted by this method to the chlorine derivative. The crude product after distillation rotated in a 0.2-dm tube at 20° C about −13.6° S. The fractionations were carried out in a fractionating column similar to the one previously described (see footnote 1). The column, 10 mm in diameter and 900 mm in length, containing glass helices 0.65 mm thick and 4.5 mm in diameter, is equivalent to a column of 22 theoretical plates.[12] The fractionations were carried out at 250-mm pressure. The capillary was supplied with dry nitrogen, and the condenser and receiver were cooled with ice water. Fractions were taken regularly from the slow fractionations, and the optical rotations were read in a 0.2-dm tube. The final product, which had a constant rotation on fractionation (−17.5° S. in a 0.2-dm tube at 20° C) was obtained after 4 months of fractionation. A quantitative chlorine determination by the Carius method gave the following results: Substance, 0.1915 g: AgCl, 0.2581 g. Calculated for $C_5H_{11}Cl$: Cl, 33.28. Found: Cl, 33.24 percent. The constants for the highest-rotating 2-chloropentane are recorded in tables 2 and 3.

(d) LEVOROTATORY 2-BROMOPENTANE

Two methods of bromination (with hydrobromic acid and with phosphorus tribromide) were studied. The bromination with hydrobromic acid was performed according to the procedure described for the preparation of the bromide derivative of the primary amyl alcohol (see footnote 1).

Forty grams of active methylpropylcarbinol ($[\alpha]_D^{20} = +13.8$) was saturated at 0° C with hydrogen bromide in a double flask. The solution was heated under reflux for 2 hours. The temperature was maintained at 70° C except for the last quarter hour, when it was allowed to rise to 85° C. The reaction product was worked up and dried as described for the bromination of the primary alcohol. The preparation was repeated for two other 40-g batches of active methylpropylcarbinol. The combined preparations were dried with potassium carbonate and distilled at 150-mm pressure to remove small amounts of the dissolved drying agent. The yield was 180 g, or 88 percent of the theoretical amount. The product was then carefully fractionated at 150-mm pressure in the column of 22 theoretical plates. A quantitative bromine determination by the Carius method on the highest-rotating fraction gave the following results: Substance, 0.2282 g: AgBr, 0.2848 g. Calculated for $C_5H_{11}Br$: Br, 52.95. Found: Br, 53.16 percent. The rotation of the product in a 0.2-dm tube at 20° C was −22.7° S.

The bromination with phosphorus tribromide was studied under various conditions with 10-g batches of active alcohol. The following method was adopted: Forty grams of active methylpropyl-

[12] F. C. Whitmore, et al., J. Am. Chem. Soc. **62**, 797 (1940).

carbinol ($[\alpha]_D^{20} = +13.6$) was cooled in ice in a flask provided with a calcium chloride tube. The flask was also connected with a burette containing 60.8 g of redistilled phosphorus tribromide, which was protected by a calcium chloride tube. The phosphorus tribromide was added slowly drop by drop to the cooled methylpropylcarbinol, and the flask was occasionally rotated. The mixture was allowed to stand at room temperature overnight, and the next morning it was heated for three quarters of an hour on the steam bath. The preparation was then cooled in an ice and salt mixture. The supernatant liquid was decanted from the sirupy phosphorus derivatives into a mixture of ice and water, under stirring. The oily bromine derivative, which separated from the ice water, was shaken repeatedly in a separatory funnel with small amounts of ice water until the wash water was neutral to litmus paper. A centrifuge was helpful for separating the product from the wash water. The preparation was separated from the adhering water and then dried over potassium carbonate. The dry preparation on distillation gave 63 g of product, or 92 percent of the theoretical amount.

The preparation was repeated several times, to yield 300 g of impure bromine derivative ranging from -22.3 to $-23.0°$ S in a 0.2-dm tube at 20° C. This product was purified by fractional distillation at 110-mm pressure until it exhibited no increase in rotation on further fractionation. The rotation of the pure product in a 0.2-dm tube at 20° C was $-25.3°$ S, which is appreciably higher than the rotation of the bromine derivative made with hydrobromic acid. A quantitative bromine determination by the Carius method gave the following results: Substance, 0.2607g: AgBr, 0.3251g. Calculated for $C_5H_{11}Br$: Br, 52.92. Found: Br, 53.07 percent. The constants found for the highest-rotating 2-bromopentane are recorded in tables 2 and 3.

(c) LEVOROTATORY 2-IODOPENTANE

The preparation of this compound was studied by two methods: The first method was by the action of hydriodic acid, and the second was by action of phosphorus triiodide on the alcohol.

The first method was performed according to the procedure described for the preparation of the iodine derivative of the primary amyl alcohol (see footnote 1) with 40 g of active methylpropylcarbinol ($[\alpha]_D^{20} = +13.8$). The yield of the product distilled at 50-mm pressure was 80 g or 89 percent of the theoretical amount. The yield of three 40-g batches was combined and carefully fractionated at 50-mm pressure. The rotation of the fraction, which did not change on further fractionation, was $-34.4°$ S in a 0.2-dm tube at 20° C. A quantitative iodine determination on this fraction by the Carius method gave the following results: Substance, 0.1647g: AgI, 0.1956g. Calculated for $C_5H_{11}I$: I, 64.09. Found: I, 64.19 percent.

The second method was applied in the following way: Ten grams of active methylpropylcarbinol ($[\alpha]_D^{20} = +13.6$) was introduced in a flask provided with a calcium chloride tube and condenser, and the flask was cooled in ice water. Forty-five grams of phosphorus triiodide of pea size, weighed off in a weighing bottle, was added to the alcohol and another 10 g of active methylpropylcarbinol was added for rinsing down the phosphorus triiodide adhering to the neck of the flask. The ice-cold mixture was gradually heated to room tempera-

ture and then heated slowly on the steam bath until all phosphorus triiodide was dissolved to form a colored solution, which required about half an hour.

The warm solution was poured on a mixture of water and cracked ice under stirring, and purified and dried as described for the first method. The yield of the distilled product was about 89 percent of the theoretical amount. Three months of careful fractional distillation at 50-mm. pressure gave a sufficient amount of the final product. The rotation of this fraction, which did not change on further fractionation, was $-34.5°$ S in a 0.2-dm tube at 20° C. This rotation is somewhat higher than that of the final product from the first method, even though a higher-rotating alcohol was used in the preparation by the first method. Thus the second method seems to be preferable. A quantative iodine determination by the Carius method gave the following results: Substance, 0.1739g: AgI, 0.2067g. Calculated for $C_5H_{11}I$: I, 64.09. Found I, 64.25 percent. The constants found for the highest-rotating 2-iodopentane are recorded in tables 2 and 3.

2. PROPERTIES OF COMPOUNDS

(a) OPTICAL ROTATION

The optical rotation was determined with a polariscope with a circular scale and double-field Lippich polarizer. Jacketed glass polariscope tubes were used. These were checked for the recorded length. For each compound investigated the rotation was determined for the two wave lengths 5892.5 and 5461 A on the circular-scale polariscope, using spectrally purified light from a sodium-vapor lamp and a mercury-vapor lamp, respectively. A third determination was made with the same sample and tube in the Bates saccharimeter, using white light and a bichromate filter, which give an effective wavelength of about 5850 A. These data are given in table 2, together with values reported by Pickard and Kenyon.

(b) SPECIFIC GRAVITY

Picnometers similar to those previously described (see footnote 1) were used for determining the specific gravity of the substances at 20° C. and a formula was applied for obtaining the specific gravity at 20°/4° in vacuum.

(c) REFRACTIVE INDEX

The values of the refractive index of the substances were determined at 20° C. with an Abbe refractometer. These data, together with calculated values of the molecular refraction, are given in table 2. A comparison of the refractivities of pentanol-2 and its halogen derivatives with the refractivities of 2-methylbutanol-1 and its halogen derivatives is given in table 6.

TABLE 2.—*Optical rotation, specific gravity, and refraction of dextrorotatory pentanol-2 and its halides of highest rotation*

Substance	Optical rotation							Specific gravity		Refractivity	
	Observed rotation, α, for—			$\frac{\alpha}{ld}$, Specific rotation at 20.0° C for—			Specific rotation by other investigators	d_4^{20} vac.	d by other investigators	Refractive index, n_D^{20}	Molecular refraction, $M=\frac{(n^2-1)}{(n^2+2)d}$
	λ 5892.5 A	λ 5461 A	White light and filter	λ 5892.5 A	λ 5461 A	White light and filter					
Dextrorotatory pentanol-2	+11.261 [a] (1 dm)	+13.238 (1 dm)	+11.290 (1 dm)	+13.901	+16.341	+13.930	[b] $[\alpha]_D^{20}=+13.70$	0.8101	[b] 0.8116	1.4056	26.704
Dextrorotatory 2-fluoropentane	+2.589 (0.2 dm)	+3.041 (0.2 dm)	+2.595 (0.2 dm)	+16.621	+19.521	+16.659		.7788		1.3520	25.032
Dextrorotatory 2-chloropentane	+5.930 (0.2 dm)	+7.026 (0.2 dm)	+5.94 (0.2 dm)	+34.072	+40.373	+34.10		.8702		1.4065	30.121
Dextrorotatory 2-bromopentane	+8.719 (0.2 dm)	+10.398 (0.2 dm)	+8.75 (0.2 dm)	+36.190	+43.161	+36.34		1.2046		1.4409	33.107
Dextrorotatory 2-iodopentane	+11.989 (0.2 dm)	+14.417 (0.2 dm)	+11.96 (0.2 dm)	+39.921	+48.015	+39.82	[b] $[\alpha]_D^{20}=+37.15$	1.5013	[b] 1.5067 for d_4^{17}	1.4951	38.478

[a] Length of tube l in decimeters.
[b] Pickard and Kenyon.

(d) BOILING POINTS AT VARIOUS PRESSURES

A Cottrell-Washburn boiling-point apparatus was used with some modifications, as described in the former publication (see footnote 1). Calibration corrections were applied to the thermometer readings. The values of the boiling points correspond to the given pressures in millimeters of mercury at 0° C and standard gravity. The results are shown in tables 3 and 7, and are shown graphically in figure 1 in comparison with the boiling-point curves for 2-methylbutanol-1 and its halides.

TABLE 3.—*Boiling points of pentanol-2 and its halides*

Pressure	Pentanol-2	2-Fluoropentane	2-Chloropentane	2-Bromopentane	2-Iodopentane
mm of Hg	°C	°C	°C	°C	°C
50	57.2	------	24.7	42.5	63.4
100	71.0	------	39.4	58.3	80.1
150	79.2	14.4	49.5	68.2	90.7
200	85.4	20.8	56.6	75.9	98.4
300	94.5	30.6	67.9	87.7	110.9
400	101.6	37.8	76.2	96.2	120.3
500	107.6	43.1	82.8	103.6	128.6
736.5 [a]	------	------	------	116.9	------
745.9 [a]	------	------	95.6	------	------
753.4 [a]	------	------	95.9	------	------
756 [a]	119.2	------	------	------	------
750 [a]	------	54.9	------	------	------
760 [b]	119.4	55.3	96.2	118.0	(c)

[a] Atmospheric pressure observed.
[b] Values calculated to normal boiling point according to Int. Crit. Tables **3**, 246 (1928).
[c] Decomposed.

3. EXPERIMENTS FOR THE CLASSIFICATION OF THE FOUR ACTIVE HALOGENOPENTANES

Before rule I could be tested it was necessary that the four active halogen derivatives be classified according to their configuration, since only derivatives of like configuration are comparable. It has been found by many investigators that starting from active secondary alcohols some reactions yield active derivatives with an inverted configuration. The subject of inversion is treated in more detail in section III.

It seemed highly probable that reaction of an optically active alcohol with analogous halogenating agents, as with HCl, HBr, and HI, or with PCl_3, PBr_3, and PI_3, should yield three active halogen derivatives of predominantly *D* configuration or three derivatives of predominantly *L* configuration. Efforts to obtain the fluorine derivative by methods analogous to those used for obtaining the other halogen derivatives were not successful, and many experiments with hydrofluoric acid under different conditions did not yield an active fluorine compound. As already described, the active 2-fluoropentane was obtained from the active bromo- or idopentane by the action of silver fluoride. Hence the above scheme could not be applied for the classification of all halogen derivatives.

It was finally realized that an analogous saponification method might be useful for deciding the question. Indeed, the following experiments led to the solution of the problem. (It is necessary to describe these experiments in detail to arrive at an interpretation.)

Twelve grams of levorotatory 2-chloropentane originally prepared

by the hydrogen chloride method and rotating −14.3° S in a 0.2-dm tube (about 83% of the rotatory power of the highest-rotating 2-chloropentane) was shaken with 200 ml of aqueous normal sodium hydroxide solution in a sealed tube at 135° C for 24 hours. After cooling the tube it was opened and the upper layer separated. The aqueous solution was neutralized with 3-normal nitric acid, filtered, and extracted with ether. The ether extract was added to the separated layer and the mixture dried and fractionated, first at ordinary pressure for separating the ether and then at 200-mm pressure. After three fractionations, in which the highest-rotating fractions were combined, a 3-g fraction was collected. This had a rotation of +4.2° S, in a 0.2-dm tube. This fraction could be identified by its positive rotation, boiling point, refractive index, and odor, and the melting point of its acid phthalate as dextrorotatory methylpropylcarbinol.

It may be calculated that by using the purest 2-chloropentane (prepared by the phosphorus trichloride method) which rotates 100/83 times higher than the 2-chloropentane used in the saponification method, a dextrorotatory methylpropylcarbinol of 4.2/0.83=5.05° S in a 0.2-dm tube could be obtained. As the alcohol originally used for the chlorination rotated +6.45° S in a 0.2-dm tube, the methylpropylcarbinol recovered from the saponification of the 2-chloropentane had the same sign of rotation as the original alcohol used for the chlorination, but the magnitude of its rotation was only 78 percent of that of the original alcohol.

The saponification of 2-bromopentane can be performed under milder conditions than those used in the saponification of 2-chloropentane. Ten grams of levorotatory 2-bromopentane of −25.2° S in a 0.2-dm tube (the purest 2-bromopentane obtained rotated −25.3° S in a 0.2-dm tube) was shaken with 140 ml of aqueous normal sodium hydroxide solution in a sealed tube at 90° C for 12 hours and worked up as described for the saponification of 2-chloropentane. A 2.5-g fraction was obtained, which exhibited an optical rotation of +4.5° S in a 0.2-dm tube. Hence the methylpropylcarbinol recovered from the saponification of 2-bromopentane had the same sign of rotation as the original alcohol used for the bromination, but the magnitude of the rotation was only 70 percent of that of the original alcohol.

The saponification of 2-iodopentane gave only a small yield of alcohol. Fifteen grams of 2-iodopentane of −33.8° S in a 0.2-dm tube (the purest 2-iodopentane rotated −34.5° S in a 0.2-dm tube) was shaken with 150 ml of aqueous normal sodium hydroxide solution in a sealed tube at 110° C for 42 hours and worked up as described for the saponification of 2-chloropentane. One and one-half grams of a fraction rotating +4.4° S in a 0.2-dm tube was obtained. The product was identified, as described for 2-chloropentane, as dextrorotatory methyl propylcarbinol. It may be calculated that by using the purest 2-iodopentane, a methylpropylcarbinol of $+4.4\times(34.5/33.8)=+4.5°$ S in a 0.2-dm tube could be obtained. Hence the methylpropylcarbinol recovered from the saponification of 2-iodopentane had the same sign of rotation as the original alcohol used for the iodination, but the magnitude of its rotation was 70 percent of that of the original alcohol.

For the saponification of dextrorotatory 2-fluoropentane, obtained by treatment of levorotatory 2-bromopentane with silver fluoride, the following procedure was used. In a preliminary experiment, 3.6 g

of slightly inpure 2-fluoropentane of +6.2° S in a 0.2-dm tube was shaken with 100 ml of half-normal aqueous sodium hydroxide solution at 95° to 100° C in a sealed tube for 12 hours. For determining the progress of the reaction, the tube was cooled and opened.

The reaction mixture consisted of a supernatant liquid and an aqueous alkaline solution. The supernatant liquid, largely unchanged fluorine derivative, was separated and without further purification showed an optical rotation of +6.6° S in a 0.2-dm tube, or an increase of 6.5 percent. The aqueous alkaline solution gave a test for bromides and was destrorotatory. Since the fluorine derivative was prepared from levorotatory 2-bromopentane, it must have been contaminated with this substance, which, on treatment with sodium hydroxide, gives sodium bromide and dextrorotatory methylpropylcarbinol. The increase in the dextrorotation of the fluoropentane was caused by removal of the levorotatory impurity, whereas the dextrorotation of the aqueous solution was caused by the presence of the dextrorotatory alcohol. In fact, a sample of the fluorine derivative obtained by microfractionations, as described elsewhere, gave an optical rotation of +7.5° S in a 0.2-dm tube, a value considerably higher than that of the material used for hydrolysis.

Unfortunately, the writer did not have a sufficient quantity of this pure material to make saponification measurements.

To effect saponification of the fluorine derivative, more vigorous treatment is necessary. The material, after treatment with sodium hydroxide, as described above, was shaken with 8 g of barium hydroxide ($Ba(OH)_2+8H_2O$) and 100 ml of water in a sealed tube at 135° C. Following the progress of the reaction, the tube was intermittently cooled and opened; a portion of the barium hydroxide solution was neutralized and filtered, and its rotation was read to determine the amount of dissolved active alcohol. The rotation of the liquid after 7 days of shaking at 135° C remained constant. The reaction product was worked up as described, and about 1 g of a levorotatory fraction of −4.1° S in a 0.2-dm tube was obtained. The optical rotation of the fraction was 64 percent of that of the alcohol originally used for the preparation. The product was identified, as described before, as the levorotatory methylpropylcarbinol. Hence the enantiomorphic levorotatory 2-fluoropentane would yield by alkaline saponification a dextrorotatory alcohol. Thus the saponification experiments show that all halogen derivatives of pentanol-2 of the same sign of rotation have the same configuration.

Another important conclusion can be drawn from the saponification experiments in conjunction with the work of Ingold and Hughes, who established (for literature, see sec. III, p. 97), that alkaline alcoholic hydrolysis of chlorine and bromine derivatives of secondary alcohols yield active alcohols with about 90 percent retention of optical activity. Thus the separation of a 60- to 70-percent optically pure alcohol by the alkaline aqueous hydrolysis indicates that the parent chlorine and bromine derivatives were about 75 percent optically pure. The low yield of alcohol from the iodine derivative may be explained by cleavage of hydrogen iodide with the formation of amylenes, but nevertheless the alcohol obtained was about 70 percent optically pure. Assuming a 90 percent retention of optical activity, the parent iodine derivative was about 80 percent optically pure. For the fluorine compound, however, another interpretation

is indicated. As already mentioned, treatment of the crude fluorine derivative with half-normal sodium hydroxide increased the dextrorotation by 6.5 percent. This increase was attributed to the hydrolysis of a small quantity of the levorotatory bromine compound present as an impurity. Of equal importance is the peculiarity that the time and temperature necessary for the saponification of the fluorine compound are considerably greater than those required for the other halogen compounds. Since the fluorine compound is exceptionally resistant to saponification, its ionization is undoubtedly low.

According to Ingold and Hughes, see footnote 13, reaction with retention of configuration usually requires ionization. Inasmuch as the fluorine compound exhibits an exceptionally low ionization, one might anticipate that replacement of the fluorine by alkaline saponification would occur almost exclusively with inversion and the optical purity of the alcohol (64%) obtained on saponification would be a measure of the optical purity of the fluorine compound. This value is questionable because the fluorine compound contained small amounts of optically inactive impurities that could be removed by fractionation and the drastic conditions of hydrolysis may have produced racemization of the alcohol formed. From these considerations the author estimates that the specific rotation of the fluorine compound obtained is approximately 65 percent ($\pm 10\%$) of that for the optically pure fluorine compound.

III. DISCUSSION OF RESULTS

1. GENERAL

As already noted, it has been established by many investigations that active secondary alcohols in which the OH group is directly attached to the asymmetric carbon atom yield with some reagents active derivatives of the same configuration and with other reagents, active derivatives with a predominantly inverted configuration. These investigations clearly showed that by displacing one of the elements directly attached to the asymmetric carbon of an optically active compound, in this case the oxygen atom of the OH group, the asymmetric grouping may be changed to its mirror image, with a more or less complete Walden inversion. But if only the hydrogen atom of the OH group is replaced and the oxygen is left undisturbed, the product of the reaction will have the same configuration as that of the original alcohol.

A part of these observations recorded in the literature which is of interest here can be condensed to the following generalizations.[13]

1. The change of an optically active secondary alcohol to a halogen derivative will involve a predominating Walden inversion of configuration.

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[13] The following references are not complete but indicate the development:
A. Werner, Liebigs Ann. Chem. **386,** 70 (1912).
J. Gadamer, J. prakt. Chem [2] **87,** 344 (1913).
J. Meisenheimer, Liebigs Ann. Chem. **456,** 121 (1927).
B. Holmberg, Ber. deut. chem. Ges. **59,** 125 (1926).
H. N. K. Rørdam, J. Chem Soc. **1930,** 2017.
H. Phillips, J. Chem. Soc. **123,** 44 (1923).
N. Meer and M. Polanyi, Z. physik. Chem [B] **19,** 164 (1932).
E. D. Hughes, C. K. Ingold, and S. Masterman, J. Chem. Soc. (**1937**) 1196, and subsequent publications. Discussion in "Physical Organic Chemistry" by L. P. Hammett, McGraw-Hill Book Co., New York, N. Y., 1940.

2. If the halogen of a derivative is displaced by another halogen, a predominating Walden inversion will take place.

3. The saponification of the halogen derivative by sodium or potassium hydroxide to the alcohol will likewise involve a Walden inversion.

In the present investigation it has been shown that dextrorotatory pentanol-2 yields, on halogenation, the levorotatory chloride, bromide or iodide; similarly, the levorotatory alcohol yields the corresponding dextrorotatory halides. Furthermore, levorotatory 2-bromopentane and 2-iodopentane on treatment with silver fluoride yield dextrorotatory 2-fluoropentane. Likewise, dextrorotatory 2-bromopentane yields levorotatory 2-fluoropentane. According to the generalizations outlined above, each of these reactions involves a Walden inversion, and it follows that dextrorotatory pentanol-2 and the four dextrorotatory halides all have the same configuration and that the levorotatory compounds are similarly related. The results of the saponification experiments reported in this paper are in harmony with this conclusion, since they show that 2-fluoro-, 2-chloro-, 2-bromo-, and 2-iodopentane of the same sign of rotation have the same configuration. The results, however, do not establish the relationship of the configuration of these compounds to that of pentanol-2.

It is appropriate to make a few remarks concerning the atomic diameter values used in this investigation.

At the beginning of this investigation only Bragg's values for the radii of ions of inorganic substances were available. Later it was found that radii of the ions of inorganic substances differ appreciably from the atomic radii of the same elements when in covalent combination. The latter radii are properly required for the author's comparisons of optical rotation and atomic dimension. However, the ratio of the differences (Cl-F):(Br-Cl):(I-Br) for the radii of the ions is nearly the same as the ratio of the corresponding differences of radii of the same elements in covalent combination. This was a rather fortunate circumstance in the development of this investigation. As soon as reliable data for the radii of the halogens in covalent combination were available, these data were used in the comparisons. Thus the ratio of the differences of Bragg's values for the ionized atoms (41:16:24) were replaced by the Goldschmidt values for the atoms in covalent combination, compiled by Wherry [14] (giving the ratio 41:14:20) or by those of de Boer and van Arkel [15] (giving the ratio 41:17:22.6) and finally by the data in Pauling's book [16] (giving the ratio 41.0:17.5:22.2). Additionally, the atomic radius data [17] derived from measurements of tetrafluoro-, tetrachloro-, tetrabromo-, and tetraiodomethane (compounds which, on account of their structure, seem to be especially suitable for arriving at reliable values) give the ratio 41.0:16.1:21.8.

Before considering the tabulated results of this investigation, attention should be given to the experimental optical-rotation values on which rules I and II, are based. The best experimental values for

[14] E. Wherry, Am. Mineral. **14,** 54 (1920).
[15] J. H. deBoer and A. E. van Arkel, Z. Physik **41,** 27 (1927).
[16] Linus Pauling, The Nature of the Chemical Bond and the Structure of Molecules and Crystals, p. 164 (Cornell Univ. Press, 1940).
[17] L. O. Brockway, J. Phys. Chem. **41,** 191, 747 (1937).
L. O. Brockway and H. O. Jenkins, J. Am. Chem. Soc. **59,** 2042 (1936).
H. A. Levy and L. O. Brockway, J. Am. Chem. Soc. **59,** 1662 (1937).
C. Finbak and O. Hassel, Z. physik. Chem. **36,** [B], 301 (1937).

establishing the ratio of the specific rotations for compounds of the first class, in which the halogen is directly attached to the asymmetric carbon atom, are obtained from the α-halogenotetraacetylglucoses. The ratio of the differences corresponding to (Cl-F):(Br-Cl):(I-Br) is 41.0:17.1:21.4. These values agree with the ratio of the differences in atomic dimension quoted above, for which the ratio 41:17:21 may be taken as an average.

If we interchange the H atom and OH group of the optically active *D*-isomer of a secondary alcohol, the active *L*-isomer of this alcohol is obtained. The structures in space of these two compounds are similar, one being the mirror image of the other. Also their physical properties are alike, but the D compound rotates the plane of polarization of light as much to the right as the other to the left. When either isomer of the alcohol is halogenated, mixtures of *D* and *L* halogen derivatives are obtained, and because of the equality of behavior of the *D* and *L* isomers and their inability to form diastereomeric derivatives, without alteration of the asymmetric center, the mixtures of these halogenated secondary alcohols are not separable by any procedure known.

In marked contrast to the halogen derivatives of the secondary alcohols, the alpha and beta modifications of a halogenoacetyl sugar are relatively easy to separate and obtain in a pure state because they are not enantiomorphs and do not have like solubilities and other physical properties. When we compare the above-mentioned optically active secondary alcohols with the optically active modifications of glucose obtained by interchange of the groups attached to carbon I, several important distinctions are apparent. By interchange of the OH group and the H atom two isomers are formed; one isomer is called α-glucose, the other β-glucose.

It has been pointed out in a detailed stereochemical study by the author[18] that the alpha and beta valences of glucose, by which the H atom and OH group are attached to asymmetric carbon I, are not equivalent in direction with respect to the structure of the entire molecule, in spite of the fact that the directions of these valences are equivalent or normal in regard to the other two valences of the first carbon. In other words, it is the ring structure which is the cause of the peculiarity.

This peculiarity in behavior is illustrated by the large difference in specific rotation between the alpha and beta sugars and derivatives, and by the fact that one pure isomer of a sugar will separate by concentrating a solution, which contains an equilibrium mixture of the isomers. This shows the difference is solubility of the two isomers. Likewise, there is considerable difference in the properties of the alpha and beta halogen derivatives. The α-halogenotetraacetylglucoses can be obtained free from the β compounds in optically pure condition and are fairly stable, whereas the β halogen derivatives are much less stable. As the four α-halogenotetraacetylglucoses differ from each other only in that one halogen (which is directly attached to an asymetric carbon atom) is replaced by another, they were excellent materials, for establishing rule I. The halogen derivatives of secondary alcohols, which are considered in this paper are not so suitable for testing rule I, because the saponification recorded and discussed

[18] D. H. Brauns, J. Am. Chem. Soc. **51**, 1821 (1929).

in this paper show the products contain *D* and *L* isomers which are not separable.

For the testing of rule II, compounds are required in which the halogen is indirectly attached (by a chain of atoms) to the asymmetric carbon. The carbohydrate derivatives are for the most part not suitable for the test because of the presence of many asymmetric centers, which, when interacting, may act partly according to rule I and partly according to rule II. However, the simple halogen derivatives containing only one asymmetric carbon atom give results in accordance with rule II, as shown for the 1-halogeno-2-methylbutanes. See footnote 1.

On account of the difficulty of obtaining the 2-halogenopentanes in the pure state, it was not possible to test the validity of rule I by the results of the present investigation. However, by assuming that chlorine, bromine, and iodine derivatives of like optical purity are obtained by treatment of pure active pentanol-2 with phosphorus trichloride, phosphorus tribromide, and phosphorus triiodide, respectively, comparisons can be made which favor the validity of rule I, and decidedly contradict the validity of rule II for this series of compounds. The preparation of 2-fluoropentane, however, required the action of silver fluoride on the active 2-bromopentane or 2-iodopentane, which reaction involves a second Walden inversion. Hence the active 2-fluoropentane might be expected to differ from the other halides in its optical purity.

On the basis of the above considerations, the following scheme was adopted for testing the relationship of the differences of specific rotations, and also the differences of the molecular rotations, to the respective differences of the atomic radii. In comparing the (Cl-F): (Br-Cl):(I-Br) ratio of the experimentally found values of the differences of specific rotations with the corresponding ratio of the differences of atomic radii (41:17:21), the (Br-Cl) rotational difference is reduced to the (Br-Cl) atomic radius difference, which is 17, and consequently the correspondingly reduced (I-Br) rotational value should not differ materially from 21. However, the correspondingly reduced (Cl-F) rotational value can be expected to differ appreciably from 41, since a second Walden inversion was involved in the preparation of the 2-fluoropentane and its optical purity may be quite different from that of the other halogen derivatives. The same scheme is also applied for testing rule II for the differences of the molecular rotations. In tables 4 and 5 the results of the investigation are recorded according to this scheme together with comments on these results.

TABLE 4.—*Comparison of the numerical relation* (Cl–F):(Br–Cl):(I–Br) *of the specific rotations of dextrorotatory 2-halogenopentanes for 5892.5 A and for 5461 A at 20° C. with the numerical relation of the differences in the radii for the respective covalent halogen bonds*

[The values given in column 5 have been obtained by multiplying the values of column 3 by a factor which will reduce the second value to 17. This value has been chosen in order to conform to the values given in column 4 for the numerical relation between the differences of neutral atomic radii, by equalizing the (Br–Cl) value.]

Halogen substituent in the 2-halogenopentane	Specific rotation	Differences between specific rotations	Relation of the differences between atomic radii	Reduced relations of differences between specific rotations
1	2	3	4	5
	For 5892.5 A			
F	+16.621			
		17.451	0.41	1.40
Cl	+34.072			
		2.118	.17	0.17
Br	+36.190			
		3.731	.21	.299
I	+39.921			
	For 5461 A			
F	+19.521			
		20.852	.41	1.27
Cl	+40.373			
		2.788	.17	0.17
Br	+43.161			
		4.854	.21	.296
I	+48.015			

TABLE 5.—*Comparison of the numerical relation* (Cl-F):(Br-Cl):(I-Br) *of the molecular rotations of dextrorotatory 2-halogenopentanes for 5892.5 A and for 5461 A at 20° C with the numerical relation of the atomic radii differences between the respective halogens.*

[The values given in column 6 have been obtained by multiplying the values of column 4 by a factor which will reduce the second value to 17. This value has been chosen in order to conform to the values given in column 5 for the numerical relation between the differences of neutral atomic radii by equalizing the (Br-Cl value.]

Halogen substituent in the 2-halogenopentane	Molecular weight	Molecular rotation in circular degrees	Differences between molecular rotations	Relation of the differences between atomic radii in angstroms	Reduced relations of differences between molecular rotations
1	2	3	4	5	6
		For 5892.5A			
F	90.138	+1498.2			
			2133.7	0.41	0.198
Cl	106.595	+3631.9			
			1834.7	.17	.17
Br	151.054	+5466.6			
			2440.1	.21	.226
I	198.058	+7906.7			
		For 5461 A			
F	90.138	+1759.6			
			2544.0	.41	.195
Cl	106.595	+4303.6			
			2216.0	.17	.17
Br	151.054	+6519.6			
			2990.1	.21	.229
I	198.058	+9509.7			

2. COMMENTS ON TABLE 4

The reduced values for the (I-Br) difference of the specific rotations for the two wavelengths agree well but do not agree so well with the required ratio of 0.21. As the purification of the bromine derivative is the most laborious, and hence the compound is probably less pure than the chlorine and iodine derivatives, it was calculated how much the specific rotation of the bromine derivative has to be changed to yield the value 0.21 instead of 0.29 for the (I-Br) reduced value. This required change for these high rotating derivatives is small, as, instead of +36.190 and +43.161 for the two wavelengths, the specific rotation of the bromine derivative should be +36.69 and +43.79. These values are only 0.5 and 0.6 circular degree higher, respectively. In other words, the bromine derivative appears to contain about 1.4 percent of inactive impurities (or only 0.7 percent of the enantiomorph). Another possibility is that the amounts of inversion which took place during the preparation of the chlorine, bromine, and iodine derivatives were not the same.

The reduced values, for the two wavelengths, of the (Cl-F) differences of the specific rotations do not agree so well with themselves or with the required ratio of 0.41. The relatively small amount of the prepared fluorine derivative (about 3 g) did not allow repeated fractionations, and the presence of amylenes or other impurities may account for the discrepency.

The large difference between the (Cl-F) value, 1.40, and the required value, 0.41, may be due to the imcompleteness of the second Walden inversion (with silver fluoride) in the preparation of the fluoride from the pure active alcohol. The value for the specific rotation of the fluorine derivative calculated on the assumption that the atomic radius relationship of rule I should hold is +28.96 for 5892.5 A. Hence the experimentally found value (+16.621) is 57.4 percent of the required value. A discrepancy of this magnitude is not surprising, since Walden inversions with silver salts are known to give variable results.[19]

It is noteworthy that the calculation of the specific rotation of the fluorine derivative from the dextrorotatory chlorine, bromine, and iodine derivatives by application of rule I gives a positive value. This indicates that the dextrorotatory fluorine derivative has the same configuration as the dextrorotatory chlorine, bromine, and iodine derivatives. This deduction is in harmony with the results of the saponification experiments. Apparently the results in table 4 support the possibility that the specific rotational values for the pure optically active 2-halogenopentanes conform to rule I.

[19] W. A. Cowdrey, E. D. Hughes, and C. K. Ingold, J. Chem. Soc. (1937) 1247. In this communication, replacements of one halogen by another are not included in the silver salt inversions, but the range of optical purity of 30 to 90 percent for the prepared compounds shows the possibility of a 57-percent value.

3. COMMENTS ON TABLE 5

The reduced values for the (I-Br) differences of the molecular rotations for the two wavelengths agree reasonably well with each other and with the value for the difference between the respective atomic radii. The (Cl-F) differences of the molecular rotations agree for both wavelengths, but they differ considerably from the value for the difference between the respective atomic radii. The calculated value for the molecular rotation of the fluorine derivative, based on the values for the molecular rotations of the other halogen derivatives and on the assumption that the relationship according to rule II should hold, is $[M]_D^{20}=-792.0$ $[\alpha]_D^{20}=-8.77$ for $\lambda=5892.5$ *A* and $[M]_{5461}^{20}=-1042.8$ $[\alpha]_{5461}^{20}=-11.57$ for $\lambda=5461$ *A*. This negative value for the rotation is in contradiction to the positive value established by the saponification experiments discussed before. Undoubtedly, the values for the molecular rotations of the pure optically active chlorine, bromine, and iodine derivatives are higher than those used in the calculation. However, if these values should be increased in approximately the same ratio the calculated optical rotation of the fluorine derivative would become even more levorotatory. Hence the experimental results for the 2-halogenopentanes in table 5 do not show a relationship between the differences in molecular rotation and the respective differences in the radii of the halogen atoms in covalent combination.

4. NUMERICAL RELATIONS FOR OTHER PHYSICAL PROPERTIES

A series of optically active halogen compounds containing a single asymmetric center was prepared primarily to furnish fundamental data for testing the relationship of the optical rotations to the atomic radii. Other physical constants were determined with the object of furnishing fundamental data, which should prove of theoretical importance. In this paper, attention will be paid to the values for the refractive indices and for the boiling points at various pressures.

Since the refractivity and boiling point of the pure *D* compound are the same as those of the pure *L* compound, these constants for mixtures containing the *D* and *L* isomers will not differ from the constants characteristic of the optically pure substances. The pentanol-2, the 2-methylbutanol-1, and the halogen derivatives of the latter were substantially pure isomers.

(a) REFRACTIVITY

In table 6 the refractive indices and the molecular refractions of the halogen derivatives of pentanol-2 are compared with similar constants for the corresponding derivatives of 2-methylbutanol-1. The results show that within the limits of error the ratios of the differences for the molecular refractions of the halogen derivatives of both amylalcohols are the same.

TABLE 6.—*Comparison of differences of the refractive indices and molecular refractions of halogen derivatives of pentanol-2 as well as those of halogen derivatives of 2-methylbutanol-1* (Cl-F):(Br-Cl):(I-Br) *with respective differences of atomic radii.*

[In the penultimate row of figures, the value 17 has been obtained by multiplying the (Br-Cl) difference by a factor which will reduce it to 17. This value has been chosen in order to conform to the (Br-Cl) value of the ratio of the difference of the neutral atomic radii (41:17:21).]

Substances	Refractivity			
	Refractive indices, n_D^{20}		Molecular refractions, $M=\frac{n^2-1}{(n^2+2)d}$	
	Pentanol-2 and halogen derivatives	2-Methylbutanol-1 and halogen derivatives	Pentanol-2 and halogen derivatives	2-Methybutanol-1 and halogen derivatives *
Alcohol	1.4056	1.4107	26.704	26.698
F derivative	1.3520	1.3576	25.032	25.011
Cl derivative	1.4065	1.4124	30.121	29.972
Br derivative	1.4409	1.4451	33.107	32.868
I derivative	1.4951	1.4977	38.478	38.041
Ratio of differences of above refractivities (Cl-F):(Br-Cl):(I-Br) and of respective atomic radii				
Ratio of differences of refractivity	0.0545:0.0344:0.0542	0.0548:0.0327:0.0526	5.089:2.986:5.371	4.961:2.896:5.173
Reducing the (Br-Cl) refractivity value to 17	27:17:27	28.5:17:29.3	29:17:30.6	29.1:17:30.4
Ratio of differences of atomic radii	41:17:21	41:17:21	41:17:21	41:17:21

* An error in calculating molecular refractions of 1-halogeno-2-methylbutanes was made in the preceding publication (p. 327, footnote 1). The correct values are recorded here.

It is readily seen from the last two rows of figures in table 6 that no correlation exists between differences of refractive indices or molecular refractivities on the one hand, and differences of respective atomic radii on the other. However, it is remarkable, that within the limits of error the reduced ratios of the differences of the molecular refractions for the two sets of halogen derivatives of the two amyl alcohols are the same, as discussed above.

(b) BOILING POINTS AT VARIOUS PRESSURES

It has been pointed out and discussed in former publications [20, 21] that the reduced differences in the boiling points at the same pressure of the (Br-Cl) and (I-Br) derivatives of 2-methylbutanol-1 have a numerical relation which agrees with that for the differences in atomic radii of the respective neutral atoms, whereas a deviation for the (Cl-F) difference exists. The measurements were made with the pure halogen derivatives uncontaminated with the enantiomorph. In this investigation we are dealing with mixtures of the *D* and *L* 2-halogenopentanes, and the data in table 7 show that also for these derivatives the (Br-Cl) and (I-Br) differences in the boiling points at the same pressure have a numerical relation which is similar to that for the differences in atomic radii of the respective neutral atoms. (The boiling points of the pure *D* compounds are the same as the boiling points of the pure *L* compounds). In figure 1, boiling point curves for 2-methylbutanol-1 and its halides (heavy lines) are compared with boiling-point curves for pentanol-2 and its halides.

[20] J. Research NBS **18**, 315 (1937) RP978.
[21] D. H. Brauns, J. Research NBS **17**, 337 (1939) RP915.

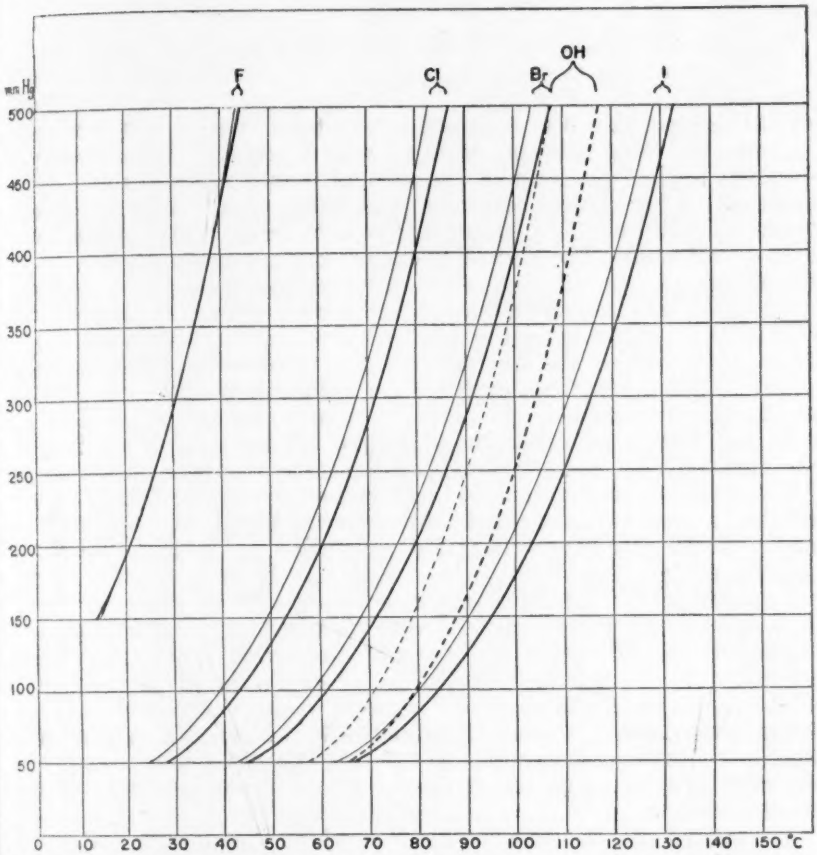

FIGURE 1.—*Boiling-point curves for 2-methylbutanol-1 and its halides (heavy lines) in comparison with boiling-point curves for pentanol-2 and its halides*

TABLE 7.—*Comparison of boiling points of 2-halogenopentanes and 1-halogeno-2-methylbutanes with the differences in the atomic radii of the hologens*

[The values given in column 3 were obtained by multiplying the values of column 2 (derived from table 3) by a factor. which will reduce the second value to 17. The value was chosen in order to conform to the (Br-Cl) value of the ratio of the differences of the neutral atomic radii (41:17:21).]

Pressure (mm of Hg)	Numerical relation between boiling point differences (Cl-F):(Br-Cl):(I-Br) for 2-halogenopentanes	Reduced numerical relation between boiling point diff (Cl-F):(Br-Cl):(I-Br) for 2-halogenopentanes	Numerical relation between differences in radii of neutral atoms (Cl-F):(Br-Cl)—(I-Br)	Reduced numerical relation between boiling point diff. (Cl-F):(Br-Cl):(I-Br) for 1-halogeno-2-methylbutanes
1	2	3	4	5
50	----:17.8:20.9	----:17:20		----:17:21.1
100	----:18.9:21.8	----:17:19.6		----:17:21.0
150	35.1:18.7:22.5	31.9:17:20.5		35.6:17:21.7
200	35.8:19.3:22.5	31.5:17:19.8	41:17:21	35.3:17:21.4
300	37.3:19.8:23.3	32.1:17:20.0		35.7:17:21.4
400	38.4:19.8:23.2	33.0:17:19.9		35.7:17:21.3
500	39.7:20.8:25.0	32.4:17:20.4		35.6:17:20.8

IV. CONCLUSIONS

The enantiomorphic modifications of pentanol-2 were prepared in the pure state, and the levorotatory isomer was converted to dextrorotatory 2-chlorine, 2-bromine, and 2-iodine derivatives. Levorotatory 2-fluoropentane was obtained by the treatment of dextrorotatory 2-bromo or 2-iodopentane with silver fluoride. The derivatives obtained by halogenation of the alcohol with phosphorus trihalides exhibited higher optical rotations than those obtained by use of the hydrogen halides. The chlorine, bromine, and iodine derivatives were obtained in a fair degree of optical purity, approximately 70 to 80 percent; the fluorine derivative, the preparation of which involved another Walden inversion, was obtained with a lower optical purity. The relative amounts of the isomeric modifications were estimated from the purity of the alcohol obtained by saponification, and the relative optical rotations of the pure 2-fluoro-, 2-chloro-, 2-bromo-, and 2-iodopentanes were calculated. It is shown that, in conformity with the results of other investigators, all halogen derivatives of pentanol-2 of like configuration have the same sign of optical rotation.

The present investigation was undertaken with the object of checking the first of two rules previously proposed by the author for correlating optical rotation with atomic dimension. The first rule states that for compounds in which the halogen is directly attached to the asymmetric carbon the differences of the specific rotations of the *D* (or *L*) compound (Cl-F), (Br-Cl), and (I-Br) have the same numerical relation as the differences of the respective atomic radii of the neutral halogen atoms. Although the difficulty of obtaining optically pure compounds, on account of incomplete Walden inversion (partial racemization) prevented an adequate check of rule I, the experimental data in no manner contradict the rule. The deviations which were observed can be plausibly explained by the incompleteness of the Walden inversion.

The second rule covers compounds in which the halogen is attached indirectly (by a chain of atoms) to the asymmetric carbon, and was applied in the comparison of the molecular rotational values of the 1-halogeno-2-methylbutanes. Rule II should not be applicable to the 2-halogenopentanes, and the results agree with this.

In contrast to this difference in optical rotation behavior between the halogen derivatives of the two active amyl alcohols (2-halogenopentanes and 1-halogeno-2-methylbutanes), it was found that these two sets of compounds are similar as to refractivity and boiling points at different pressures.

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TENSILE AND OTHER PROPERTIES OF CONCRETES MADE WITH VARIOUS TYPES OF CEMENTS

By Louis Schuman and John Tucker, Jr.

ABSTRACT

An improved tensile-test method for concrete has been developed. Cylindrical specimens, 4 by 16 inches, are used, and loads are applied at both ends through threaded rods embedded in a rich mortar. Compressive- and tensile-strength and stress-strain determinations have been made for concretes made with various types of cements, including cements with aerating agents, and with various aggregates. At early ages, strengths for moderate-heat cements were but slightly lower than for normal cements, whereas those for high-early-strength cements were considerably higher. Tensile strengths usually attained maximum values between 7 days and 3 months. Tensile strengths depended on compressive strengths, type of cement, type of aggregate, and on sand-gravel ratios.

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I. INTRODUCTION

The tensile strength of concrete is an important factor in its resistance to fracture by bending, shrinkage, freezing and thawing, or differential expansion. Although a number of investigations on both tensile and flexural properties of concretes have been reported, no comparative data on the effect of the newer types of cement are available. The present investigation was undertaken to develop a more satisfactory tensile-test method for concrete than has been used in the past, and particularly to secure data on the compressive, tensile, and stress-strain properties of concretes made with various types of cement, including cements with aerating agents. In addition, the effect of varying the sand-gravel ratio or the type of coarse aggregate on the tensile strength of concrete was studied.

II. TENSILE-TEST METHODS

1. PREVIOUS METHODS

Unlike the compressive test, which has been standardized [1],[1] tensile tests of concrete have been made by a number of different methods [2 to 9]. The dimensions of the specimens have ranged from 12 to 39 in. in length, and from 3 to 6 in. square, or in diameter. The shapes of the specimens have also varied, depending on the method of applying the load. Some investigators [4, 7, 8, 9] used specimens with enlarged ends to which grips were applied. Others [5, 11] cast cylindrical specimens and used grips clamped or wedged to the ends of the test pieces. Besides the use of unduly large specimens, some of these methods are open to objection because of the way the specimens break. Specimens with enlarged ends often break at the junction of the enlarged and reduced portions. Cylindrical specimens, on the other hand, may break near or within the grips [5].

2. IMPROVED METHOD

The type of specimen and method of loading developed in the present investigation are believed to overcome these objections. In principle, the method is similar to that used in the tests of Talbot and of Withey [2, 3].

The new feature of the improved method of determining tensile strength is the application of the load to the concrete under test through threaded rods embedded in a rich mortar molded on the ends of the cylindrical test specimens (fig. 1). The preliminary tests with this method showed that the specimens always broke well within the concrete under test, and that a bond stress of 700 lb/in.^{2} of embedded rod area could be safely used. Thus, for the rods in figure 1, giving about 12 in.^{2} of bond area, no failure in bond or at the ends of the rods will occur for loads up to 8,000 lb., equivalent to a tensile stress in the concrete of 600 lb/in.^{2}.

A few tests were made to compare the new method with certain of the older methods. With a 1:4.5 concrete, six specimens were molded and tested by the new method, and six by each of the following methods:

[1] Figures in brackets indicate the literature references at the end of this paper.

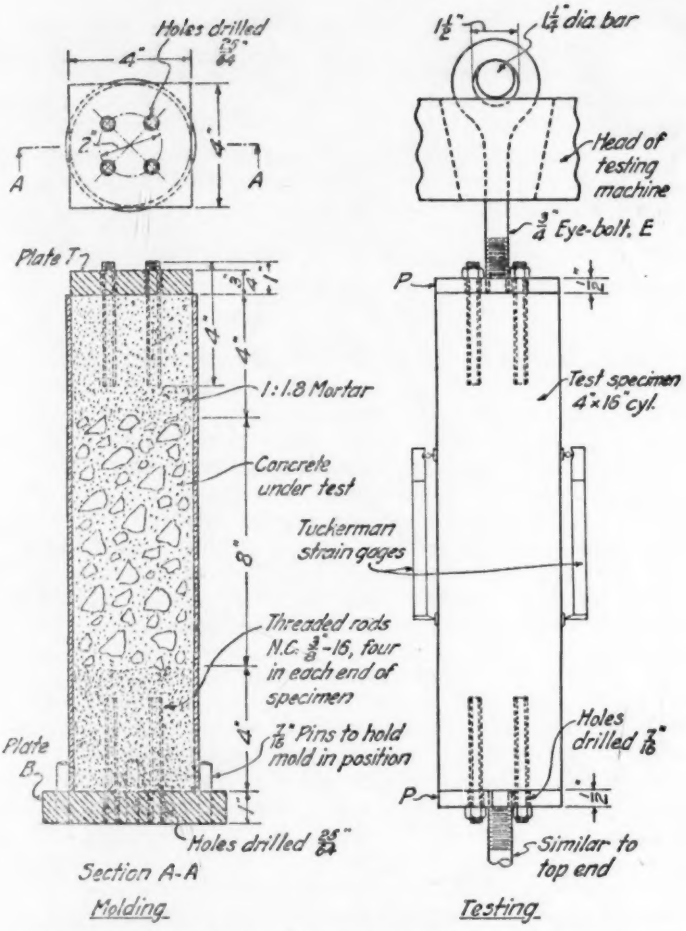

FIGURE 1.—*Method of molding and of testing tensile specimens.*

1. Specimens 4½ in. in diameter, with splayed ends; space between grips and specimens filled with a plastic material. Method similar to Johnson's [4].

2. Specimens 3 in. in diameter, plain rods of unequal lengths embedded directly in the ends of the concrete; load applied through plates attached to the rods at each end of the specimen. Method similar to Davis's [6].

By method 1, the values of tensile strength obtained ranged from 350 to 420 lb/in.², with an average of 380 lb/in².

By method 2, the values ranged from 245 to 380 lb/in.², with an average of 320 lb/in.².

The values obtained by the new method ranged from 400 to 420 lb/in.², with an average of 430 lb/in².

The higher value obtained by the new method is believed to indicate a more uniform stress distribution across the section of failure than with the other methods.

A 1:3 mortar made with standard Ottawa sand, as used in cement testing (Federal Specification SS–C–158a), was also tested. Mortar from the same batch was molded into standard briquets and into cylinders of the type used for the new concrete tensile-test method. At 7 days the average strength for three briquets was 295 lb/in.^{2}, and for three cylinders, 360 lb/in.^{2}. At 28 days the averages were 415 and 565 lb/in.^{2}, respectively. The cylinders thus gave 22 and 36 percent higher strengths than the briquets.

III. MATERIALS AND TESTS

1. CEMENTS

The cements used in the tests were five normal portland cements, including a white portland, designated N–1 to N–5, table 1; four moderate-heat portland cements, M–1 to M–4; three high-early-strength portland cements, S1–S3; two portland-pozzolan cements, P–1 and P–2; one high-alumina cement, L; and eight portland cements with rosin, tallow, or vinsol resin as aerating agents ground in at the mill, table 2, C to J. One cement was tested both without and with a soluble aerating agent, sodium lauryl sulfate, added in the laboratory, so that a comparison could be obtained between aerated and unaerated concretes for the same cement, table 2, A and B. Each of the cements within a type was from a different mill.

2. MIXES AND TESTS

For the tests with the various cements, the aggregates were Potomac River sand and gravel. The fineness modulus of the sand varied from 2.6 to 2.8. The maximum size of the gravel was ¾ in., with 34 percent passing the ⅜-in. sieve and 100 percent retained on the No. 4.

For all but the aerated concretes, two mixes were tested with each of the cements, a 1:2.4:3.6 by weight (mix *B*, 1:6) and a 1:1.75:2.75 by weight (mix *H*, 1:4.5). Only mix *B* was used in the tests on aerated concretes. Compressive and tensile specimens, in triplicate, were made for test at 3, 7, and 28 days, 3 months, and 1 year, except that for the high-early-strength and high-alumina cements tests at 1 day were added and the 3-month compressive tests omitted. In addition to the compressive and tensile tests, stress-strain relations were determined on the concretes made with the cements containing aerating agents.

To study the effect of varying the aggregate proportions, a series of tensile tests was made in which the sand-gravel ratio in a 1:6 mix with the Potomac aggregates was varied. These tests were made with each of three different portland cements. The proportions varied from 1:1.9:4.1 to 1:3.4:2.6, and the water content was varied correspondingly to maintain approximately equal flow values. Tensile strengths at 3 and 28 days were determined.

Another series of tests was made to study the effect of a few different coarse aggregates. For this series, a single cement with Potomac sand was used. The coarse aggregates were Potomac gravel, a New York limestone, an Ohio slag, a Pennsylvania trap-rock, and a New York granite. Two mixes were tested, except for the granite and trap-rock, for which there was only sufficient material

for one mix. For the Potomac gravel, the mixes were the same as for the tests of the various cements. For the other coarse aggregates, the cement-sand ratios were the same as for the Potomac, but the proportions of coarse aggregate were varied with the bulk specific gravities so as to give approximately equal yields of concrete. The water contents were varied to give equal flow values, except in one case where the Potomac gravel was tested in a mix with the same *C/W* ratio as was used for the other aggregates. Tests were made on the 1:6 mixes at 7 and 28 days, and on the 1:4.5 mixes at 3 and 28 days (table 4).

In all of the above tests, a set of three specimens was tested at each test age. The cements used in the series of tests with various aggregates and sand-gravel ratios were of different brands or lots from those mentioned in section III–1.

IV. PREPARATION OF TEST SPECIMENS

1. COMPRESSIVE SPECIMENS

All mixing and molding of the test specimens was done in a laboratory maintained at 70°±3° F. The 6 by 12 in. compression cylinders were prepared by ASTM method C39–39 [1]. The mixing time was generally 2 minutes. It was found, however, that the 2 minutes of hand mixing was not sufficient to incorporate substantial amounts of air where the cements contained aerating agents. Hence these concretes were mixed for 3 minutes.

Flow values were determined by ASTM method C124–39, [1, P. 337] except that the mold was filled without rodding, and the table dropped 15 times through a distance of ½ in. The cement-water ratio was generally 1.66 for the 1.6 mix and 2.05 for the 1:4.5 mix (6.8 and 5.5 gal/bag, respectively) but was varied somewhat to maintain approximately equal flows.

The tops of the compression cylinders were capped with neat-cement paste, not less than 4 hours after molding them. For the slower-setting cements, capping the cylinders in less than 4 hours after molding, results in distortion of the caps.

The cylinders were left in the laboratory for 24 hours, then removed from the molds and stored in the damp room at 70°±3° F until tested.

2. TENSILE SPECIMENS

The method of molding the tensile-test specimens can be seen in figure 1. The ⅜- by 4-in. threaded rods were held in position by the holes in the bottom plate, *B*, and the mold was held in place by the pins in the plate. The mortar for the ends of the tensile specimen consisted of the test cement and sand, 1:1.8 by weight, with a *C/W* (cement-water) ratio of 2.4 to 2.6. Sufficient mortar for a set of three tensile specimens was mixed in a pan, by hand. The lower ends of three specimens were then molded by rodding the mortar, in two layers, each about 2 in. in depth, so as to extend about 1 in. above the ends of the threaded rods, figure 1. Concrete, sufficient for 3 specimens, was then mixed and placed in the mold in two 4-in. layers, each rodded with 20 strokes of the tamping rod. The remaining mortar was then placed and rodded into the top of each mold.

The plates, *T*, were then placed in position over the tops of the molds and the threaded rods inserted through the holes, with some turning and vertical motion to secure good bond to the mortar. The rods extended 1 in. beyond each end of the specimen to provide for the plates used in testing, figure 1, and had a length of 3 in. embedded in the mortar. The space between the ends of the rods and the concrete contained a mixture of mortar and concrete.

The threaded rods were lightly greased before molding into the tensile specimens, and could be readily removed by splitting the cylinders. The same rods were then used in other specimens.

The tensile specimens were stored and cured under the same conditions as the compressive specimens.

V. TEST METHODS

1. COMPRESSIVE AND TENSILE TESTS

The compressive tests were made in a 300,000-lb. hydraulic testing machine at a rate of loading of 1,000 lb./in.^{2} a minute. The tensile tests were made in a 100,000-lb. beam-and-poise, screw-driven testing machine, operating at an idle head speed of 0.05 in. per minute. One-half-inch plates, *P*, figure 1, were attached to the specimen, and eye-bolts, *E*, attached to the plates. The loads were applied through bars passing through the eye of each bolt and resting against the testing-machine heads.

2. STRESS-STRAIN DETERMINATICNS

Strains were measured with two Tuckerman optical strain gages [10], extended to a 6-in. gage length. The gages were seated on thin brass plates attached with plaster of paris to the specimen, and were diametrically opposite each other. An intitial stress of 50 or 100 lb/in.^{2} for the compressive specimens, or of 10 lb/in.^{2} for the tensile specimens, was applied for the first readings. Rates of loading were the same as for the strength tests, but the loads were maintained as nearly constant as possible while the gages were being read. Strains were measured to less than one-millionth.

However, because of the difficulty of maintaining constant loads with the hydraulic machine, any load increment may be in error by as much as 400 lb, equivalent to 14 lb/in.^{2} on a 6- by 12-in. cylinder, corresponding to a strain error of 4×10^{-6} for a modulus of 3.5×10^6 lb/in.^{2} This error is less than 2 percent of the strain at a stress of 1,000 lb/in.^{2}, and correspondingly lower at higher stresses.

VI. TEST RESULTS AND DISCUSSION

1. CONCRETES MADE WITH VARIOUS CEMENTS

(a) GENERAL

The test results for two concrete mixes with the various cements, and for the aerated concretes, are given in tables 1 and 2. For any one specimen of a set of three, the deviation from the average was usually less than 10 percent, both for the tensile and compressive specimens. Practically none of the tensile fractures occurred at a distance of less than 2 in. from the ends of the threaded rods, and about 80 percent of the fractures were within 2 in. of the middle of the specimen.

TABLE 1.—*Compressive and tensile strengths of concretes made with various types of cement*

TABLE 1.—*Compressive and tensile strengths of concretes made with various types of cement*

[Each strength value is the average for 3 test specimens]

Type of cement	Identification [1] number	C/W [2] by weight	Flow	Compressive strength (lb/in.²) at—						Tensile strength (lb/in.²) at—					
				1 day	3 day	7 day	28 day	3 mo.	1 yr.	1 day	3 day	7 day	28 day	3 mo.	1 yr.
MIX *B*. 1:6 BY WEIGHT															
			%												
Normal	N-1	1.66	48	.....	1,490	2,260	4,440	5,030	5,540	.....	170	240	355	360	355
Do	N-2	1.66	53	.....	1,420	2,260	4,300	5,440	5,960	.....	170	230	415	430	450
Do	N-3	1.66	48	.....	1,520	2,730	4,800	5,420	5,800	.....	170	255	395	365	370
Do	N-4	1.66	45	.....	1,490	2,820	5,030	5,900	6,160	.....	200	285	440	425	400
Normal (white)	N-5	1.66	55	.....	1,560	2,660	4,560	5,040	5,960	.....	195	290	425	385	430
Average				.....	1,490	2,550	4,630	5,370	5,880	.....	180	260	405	395	400
Moderate heat	M-1	1.66	57	.....	1,030	1,760	3,550	5,560	6,530	.....	135	200	355	435	470
Do	M-2	1.66	46	.....	1,040	2,170	4,560	5,250	6,480	.....	150	260	395	375	435
Do	M-3	1.66	50	.....	1,440	2,510	4,420	5,330	6,340	.....	195	285	385	405	445
Do	M-4	1.66	50	.....	1,380	2,350	4,480	5,460	5,800	.....	190	275	430	405	405
Average				.....	1,220	2,200	4,250	5,400	6,290	.....	170	255	390	405	440
Portland-pozzolan	P-1	1.66	47	.....	800	2,060	4,670	5,530	5,860	.....	135	230	415	370	410
Do	P-2	1.55	52	.....	1,140	1,910	3,000	3,900	4,270	.....	160	235	315	310	355
Average				.....	970	1,980	3,830	4,710	5,070	.....	150	230	365	340	380
High-early strength	S-1	1.66	49	1,020	2,580	3,690	5,420	.....	5,690	150	300	430	505	410	435
Do	S-2	1.59	54	1,150	2,730	3,980	5,500	.....	5,280	160	300	435	460	365	365
Do	S-3	1.66	44	1,020	3,310	4,280	6,150	.....	5,970	155	365	445	480	420	390
Average				1,060	2,870	3,980	5,690	.....	5,650	155	320	435	480	400	395
High alumina	L	1.66	58	4,030	4,270	4,980	5,510	.....	4,130	295	330	290	275	.....	220

See footnotes at end of table.

TABLE 1.—*Compressive and tensile strengths of concretes made with various types of cement*—Continued

Type of cement	Identification [1] number	C/W [2] by weight	Flow	Compressive strength (lb/in.²) at—						Tensile strength (lb/in.²) at—					
				1 day	3 day	7 day	28 day	3 mo.	1 yr.	1 day	3 day	7 day	28 day	3 mo.	1 yr.
MIX *H.* 1:4.5 BY WEIGHT															
			%												
Normal	N-1	2.05	49	.....	2,210	3,360	5,270	6,010	6,650	.....	265	390	465	425	455
Do	N-2	2.05	52	.....	2,080	3,230	5,360	6,790	7,910	.....	250	360	530	530	525
Do	N-3	2.05	53	.....	2,380	3,990	5,900	6,610	7,700	.....	265	435	470	505	490
Do	N-4	2.05	52	.....	2,280	4,070	6,620	7,050	8,030	.....	300	410	515	500	485
Normal (white)	N-5	2.05	59	.....	2,390	3,450	5,290	6,130	7,480	.....	250	350	460	465	515
Average				.....	2,270	3,630	5,690	6,520	7,550	.....	265	390	490	485	495
Moderate heat	M-1	2.05	61	.....	1,670	2,780	5,020	6,880	8,400	.....	230	310	520	535	595
Do	M-2	2.05	47	.....	1,760	3,250	5,880	6,750	7,690	.....	235	400	490	485	520
Do	M-3	2.05	42	.....	2,100	3,620	5,230	6,580	7,820	.....	285	440	480	465	545
Do	M-4	2.05	40	.....	2,110	3,390	5,860	6,320	7,490	.....	300	425	515	475	460
Average				.....	1,910	3,260	5,500	6,630	7,850	.....	260	390	500	490	530
Portland-pozzolan	P-1	2.05	50	.....	1,390	2,520	5,610	6,460	6,780	.....	210	310	480	470	545
Do	P-2	1.88	54	.....	1,760	2,880	4,340	4,960	5,510	.....	225	315	415	385	435
Average				.....	1,580	2,700	4,980	5,710	6,140	.....	220	310	445	430	490
High-early strength	S-1	2.05	42	1,750	3,580	4,680	5,770	.....	7,370	210	390	525	530	495	510
Do	S-2	1.96	47	1,790	3,600	5,070	6,120	.....	7,150	240	420	520	495	445	415
Do	S-3	1.94	47	1,540	3,770	4,710	6,300	.....	7,290	205	410	465	500	490	495
Average				1,690	3,650	4,820	6,060	.....	7,270	220	405	505	510	475	475
High alumina	L	2.05	51	4,590	.....	5,250	6,310	.....	4,560	375	435	445	350	.....	305

[1] Numbers following the type letter signify different brands of that type.
[2] Ratio of weight of cement to weight of water in the mix.

TABLE 2.—*Properties of aerated concretes (mix B: 1:6 by weight)*

TABLE 2.—*Properties of aerated concretes (mix B: 1:6 by weight)*

[Each strength or modulus value is the average for 3 test specimens]

Cement [1]	Aerating agent	C/W by weight	Flow	Weight of fresh concrete	Compressive strength at—					Tensile strength at—				Modulus of elasticity/10^6 at 28 days	
					3 day	7 day	28 day	3 mo.	1 yr.	3 day	7 day	28 day	3 mo.	Compression	Tension
			%	*lb/ft.*^{3}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}
A	None	1.66	45	146.2		2,580	4,650			220	305	385	380	4.1	4.0
B [2]	Sodium lauryl sulfate	1.82	46	137.8		2,050	3,720				290	345		3.6	3.3
C	Rosin	1.66	45	141.2	1,150	1,970	3,830	3,960	4,540	190	270	360	355	3.6	3.5
D	Tallow	1.74	45	139.9	1,230	2,190	3,060	4,200	4,640	220	275	330	385	3.6	3.5
E	do	1.66	43	141.6	1,470	2,340	4,210	4,130	4,520	210	330	325	310	3.7	3.6
F	do	1.66	46	139.6	1,210	2,180	3,430	3,790	3,940	185	270	355	325	3.4	3.3
G	do	1.68	46	143.2	1,440	2,500	4,070	4,860	5,310	210	300	390	415	3.7	3.6
Average				141.1	1,340	2,400	3,690	4,250	4,600	210	290	350	360	3.6	3.5
H	Vinsol resin	1.74	46	142.3	1,710	2,710	4,310	4,780	5,340	255	320	390	405	3.9	3.6
I	do	1.70	45	143.6	1,920	2,960	4,380	5,050	5,520	250	320	350	390	3.9	3.8
J	do	1.77	47	141.2	1,350	2,510	3,960	4,630	5,560	190	250	315	410	3.7	3.7
Average				142.4	1,660	2,730	4,160	4,820	5,470	230	295	350	400	3.8	3.7

[1] These cements were of different lots or brands from those listed in table 1.
[2] Normal cement A, with 0.05 percent of a soluble aerating agent added in the laboratory.

(b) VARIATION IN STRENGTH FOR CEMENTS WITHIN A TYPE

There was sometimes considerable variation in strength for different cements within a type, as, for instance, for the two portland-pozzolan cements, table 1. However, for both the normal and the high-early-strength portland cements, the variation of either compressive or tensile strength from the average for the type was, at any age, less than 16 percent. Somewhat greater variation is found among the moderate-heat cements. The aerated concretes, table 2, also show considerable variation, as is to be expected from the effect of the various aerating agents on the weight per cubic foot of concrete. This varied from 138 to 144 lb compared with 146 lb for the unaerated concrete made with cement A.

(c) VARIATION IN STRENGTH WITH TYPE OF CEMENT

In figure 2 the average strengths for each type of cement are plotted against age. For both mixes the strengths for the high-early-strength cements, both in tension and compression, for ages up to and including 28 days are higher than for all others, except for the 1:4.5 mix with the high-alumina cement.

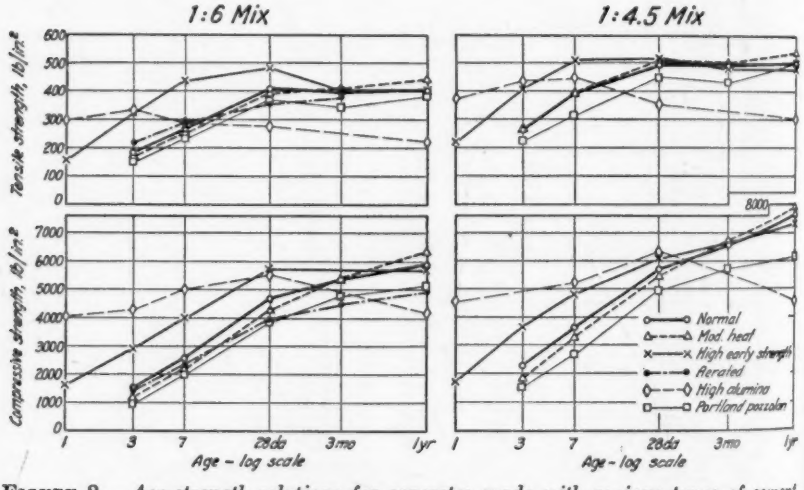

FIGURE 2.—*Age-strength relations for concretes made with various types of cement.*

Strength values are averages for each type of cement.

In figure 2 it may be seen that to attain an average compressive strength of 4,600 lb/in.² requires about 28 days for the normal cements, 1:6 mix, and about 12 days for the high-early-strength cements in the same mix. Similarly, for an average tensile strength of 400 lb/in.², attained by the normal cements in 28 days, only about 6 days are required by the high-early-strength cements. A similar relation is seen for the 1:4.5 mix. Thus it appears that the use of a high-early-strength cement is of greater advantage in securing high early tensile than high early compressive strength.

For both the 1:6 and 1:4.5 mixes, the moderate-heat cements gave strengths up to about 28 days somewhat lower than those for the normal cements. After 3 months there is a tendency for the moderate-

heat cements to give slightly higher strengths, both in tension and compression, than the normal cements.

The aerated concretes, represented only by the 1:6 mix, show a wider range of strengths than the other types, as is to be expected both from the variety of aerating agents used and the number of cements tested. Although the tensile strengths at the early ages are on the average slightly higher than those for the normal cements, at the later ages there is a tendency for both tensile and compressive strengths to fall below those for the normal cements.

The two portland-pozzolan cements differed widely from each other, table 1. For one of these, the strengths up to 7 days were within the range of those for the moderate-heat portland cements; however, after 7 days the strengths for this cement were appreciably lower than those given by the moderate-heat cements. The other portland-pozzolan cement gave relatively low strengths only up to 3 or 7 days, but thereafter the strengths were within the range of those for the normal cements.

The high-alumina cement is unique in showing very high early strength (at 3 days or less). However, compressive strengths began retrogressing some time after 28 days and tensile strengths some time after 3 days for the 1:6 mix and 7 days for the 1:4.5 mix, so that at 1 year the strengths for this cement, both in tension and compression were no higher than at 1 day, and were less than for any other type of cement.

(d) EFFECT OF AGE OF CONCRETE ON STRENGTH

While the concretes made with two of the three high-early-strength cements (table 1) showed slight retrogression in compressive strength for the 1:6 mix after 28 days, those made with all the other types of cement except the high-alumina showed continued increase in compressive strength, for both mixes, up to a year. Tensile strengths, however, usually reached a maximum value at some time between 7 days and 3 months, followed by retrogression (table 1). The maximum tensile strengths obtained for the 1:6 mix with the high-early-strength cements were higher than those for the same mix with the other cements (fig. 2). However, the high-early-strength cements showed relatively large retrogressions, and, at 3 months and thereafter, gave about the same tensile strengths as the normal cements. For the 1:4.5 mix, two of the three high-early-strength cements gave maximum values at 7 days, but these were no higher than those attained by other cements at later ages. One of the three cements (S–2, table 2) of this type showed large retrogression for this mix.

The average strengths for the moderate-heat cements at 1 year were the highest for all cements tested. For the normal and portland-pozzolan cements, any retrogression between 28 days and 3 months was usually followed by some recovery, so that, at 1 year, the tensile strengths are never more than 10 percent lower than at 28 days, and are often higher.

The retrogressions in tensile strengths of concretes after certain ages may be due to retrogression in mortar tensile strengths, since such retrogression occurs with mortar briquets. The retrogressions may also be due in part to decrease in bond strength between gravel and mortar. Considerable gravel fracture was noted at 28 days,

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and there was some tendency for the percentage of gravel fractured to decrease with age thereafter.

(e) EFFECT ON STRENGTH OF CEMENT CONTENT OF CONCRETE

In general, the concrete with the higher cement content (1:4.5 mix) gave higher strength, for all cements, than the 1:6 mix, at correspond-

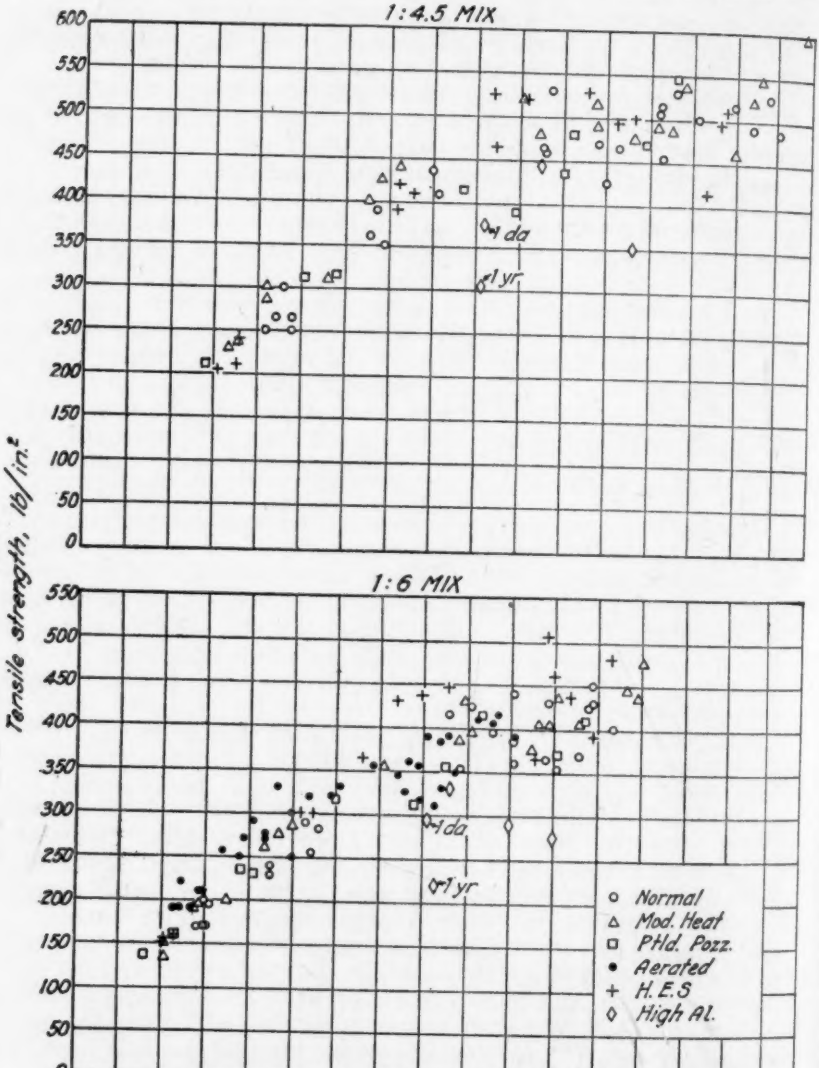

FIGURE 3.—*Relations between compressive and tensile strengths.*

ing ages. However, there are some noteworthy differences between the rates of increase of tensile and of compressive strength. For instance, comparing the 1:4.5 mix with the 1:6 mix for the normal

cements, we see (fig. 2) that to attain an average compressive strength of 4,600 lb/in.^{2} requires 28 days for the 1:6 mix and about 14 days for the 1:4.5 mix. Similarly, the average tensile strength for the 1:6 mix is about 400 lb/in.^{2} at 28 days, and this is attained by the richer concrete in only 8 days. A similar relation for the two mixes with the high-early-strength cements can also be seen. Thus the use of the richer mix is of greater advantage in securing high early tensile than high early compressive strength.

(f) RELATIONS BETWEEN COMPRESSIVE AND TENSILE STRENGTHS

In figure 3 the tensile strengths have been plotted against the corresponding compressive strengths for each of the two mixes, for all ages. It is seen that there is a fair relation between compressive and tensile strengths.

For equal compressive strengths, tensile strengths tend to be somewhat higher for the 1:4.5 mix than for the 1:6. There is also a tendency for the aerated concretes to have higher tensile strengths for equal compressive strengths up to about 2,500 lb/in.^{2} than the other 1:6 concretes. The high-alumina cement shows appreciably lower tensile strength, for equal compressive strengths, than the other cements.

(g) STRESS-STRAIN RELATIONS

The stress-strain curves for the 1:6 concretes made with cements containing aerating agents and for cement A (the normal cement used for the tests with a soluble aerating agent), are shown in figures 4 and 5. The cements are arranged in the order of compressive strengths.

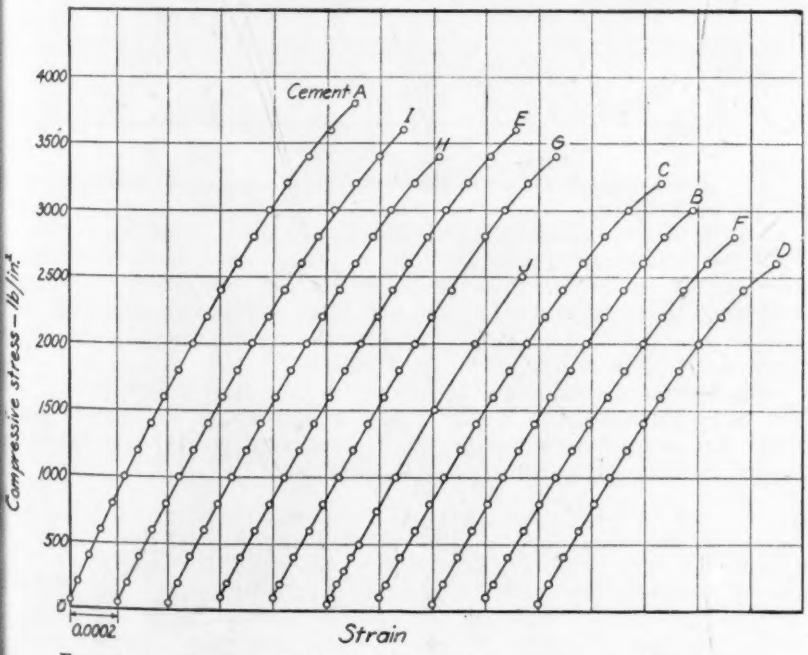

FIGURE 4.—*Stress-strain relations for 1:6 concretes in compression.*

For the compressive specimens the difference in strain values for opposite sides of the same specimen was usually less than 60×10^{-6} for stresses up to three-fourths of the breaking stress; this corresponds to a stress difference of 250 lb/in.^{2} or less. For the tensile specimens the corresponding difference in strain values was usually less than 10×10^{-6}, equivalent to a stress difference of 40 lb/in.^{2} or less. The deviation of the strain for one specimen from the average for a set of three was only about 20×10^{-6} for the compressive tests, and about 3×10^{-6} for the tensile tests.

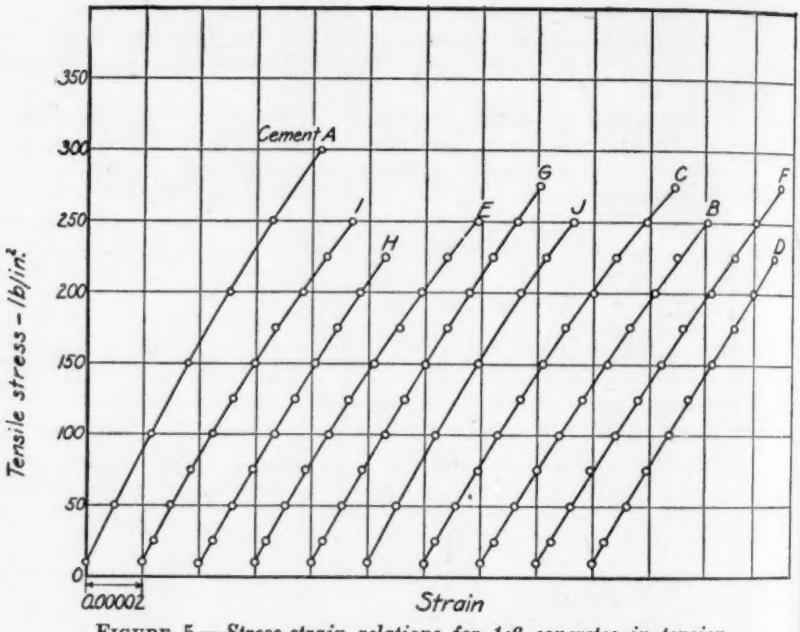

FIGURE 5.—*Stress-strain relations for 1:6 concretes in tension.*

Values of the initial tangent moduli, obtained from the curves of figures 4 and 5, are given in table 2. In general, both compressive and tensile moduli increase with increasing compressive strengths and weights per cubic foot of the concretes. The compressive moduli varied from 3.4 to 4.1×10^6 lb/in.^{2}, and the tensile moduli varied from 3.3 to 4.0×10^6 lb/in.^{2}.

The tensile moduli show no relation to the tensile strengths. For the same concrete, the tensile modulus appears to be slightly lower than the compressive modulus. Similar but greater differences in secant moduli have been noted by Johnson [7].

2. EFFECT OF SAND-GRAVEL RATIO AND OF VARIOUS COARSE AGGREGATES ON TENSILE STRENGTH

(a) SAND-GRAVEL RATIO

The results of the tests with Potomac River aggregate in which the sand-gravel ratio was varied are given in table 3. It is seen that for all three cements, K, L, and M, the 1:2.9:3.1 mix gave the highest tensile strengths, both at 3 and 28 days (except in one instance where the strength is within 5 lb of the maximum) despite the fact that the water requirement was higher than for the 1:2.4:3.6 mix. The 1:1.9:4.1 mix, which was deficient in sand, gave low tensile strengths despite the high C/W ratios. The 1:3.4:2.6 mix, which was definitely oversanded, had both a high water requirement and low tensile strengths.

TABLE 3.—*Tensile strength of concretes with various sand-gravel ratios*

[Each strength value is the average for 3 test specimens]

Mix (proportion by weight) [1]	*C/W* by wt	Water gal/bag	Flow %	Tensile strength at—	
				3 days lb/in.²	28 days lb/in.²
CEMENT K [2]					
1:1.9:4.1	1.73	6.5	48	145	325
1:2.4:3.6	1.66	6.8	49	180	380
1:2.9:3.1	1.53	7.4	52	190	430
1:3.4:2.6	1.42	7.9	49	160	380
CEMENT L [2]					
1:1.9:4.1	1.73	6.5	49	180	420
1:2.4:3.6	1.66	6.8	44	220	465
1:2.9:3.1	1.53	7.4	43	240	460
1:3.4:2.6	1.42	7.9	55	175	415
CEMENT M [2]					
1:1.9:4.1	1.66	6.8	47	145	320
1:2.4:3.6	1.61	7.0	45	160	350
1:2.9:3.1	1.48	7.6	45	170	365
1:3.4:2.6	1.33	8.5	53	140	325

[1] Made of cement, sand, and ¾-in. gravel.
[2] Normal portland cements of different brands or lots from those listed in tables 1 and 2.

(b) COARSE AGGREGATES

In table 4 are given the results of the tests with various coarse aggregates.

For both mixes, Potomac gravel gave the highest strengths of any aggregate at 3 and 7 days, even when the lower *C/W* ratio was used. At 28 days the Potomac gravel gave higher strength than the other aggregates in the 1:6 mix, but the New York limestone gave the highest value for the 1:4.5 mix. Both the trap-rock and granite gave fairly low values for the 1:4.5 mix at 28 days, whereas the slag, which had about the same bulk specific gravity as Potomac gravel, gave a rather high value.

TABLE 4.—*Tensile strengths of concretes with various coarse aggregates*

[Each strength value is the average for 3 test specimens.]

Coarse aggregate	Proportions[1] by dry wt	C/W by wt	Water	Flow	Tensile strength[2] at—		
					3 days	7 days	28 days
			gal/bag	*percent*	*lb/in.²*	*lb/in.²*	*lb/in.²*
Potomac gravel	1:2.4:3.6	1.66	6.8	41	----	280	445
Do	1:2.4:3.6	1.48	7.6	78	----	240	385
N. Y. limestone	1:2.4:4.0	1.48	7.6	43	----	175	315
Ohio slag	1:2.4:3.6	1.48	7.6	40	----	195	380
Potomac gravel	1:1.75:2.75	2.05	5.5	40	315	----	445
N. Y. granite	1:1.75:2.80	1.88	6.0	35	205	----	405
N. Y. limestone	1:1.75:3.00	1.88	6.0	40	205	----	525
Pa. "trap-rock"	1:1.75:3.00	1.88	6.0	39	230	----	380
Ohio slag	1:1.75:2.75	1.88	6.0	35	230	----	460

[1] The amounts of coarse aggregates were varied in proportion to their bulk specific gravities.
[2] The normal cement used for these tests was different from those of the previous tables.

For the 1:4.5 mix, the 28-day tensile strengths vary over a range of almost 1.4 to 1, depending on the coarse aggregate. Kellerman [11] has noted a similar variation, as well as the fact that relative tensile strengths for different aggregates depend on the richness of the mix. The New York limestone, for instance, gave the highest 28-day tensile strength in the rich mix, but the lowest in the leaner mix. A possible explanation is, that with some aggregates, such as Potomac gravel, the bond strength between mortar and coarse aggregate may be relatively high; this would tend to give relatively high strength at the early ages, when little of the coarse aggregate is broken. With the New York limestone, on the other hand, bond strength may be relatively low with the leaner mix, resulting in low strength at the early ages; but this aggregate may itself have higher strength and, when used in a richer mix tending to cause failure of the aggregate, the concrete strength may be rather high.

Some of the fractured sections are shown in figures 6 and 7. Figure 6, *A*, is a section of 1:6 aerated concrete made with Potomac gravel, tested at 3 days. This concrete had an average tensile strength of 220 lb/in², and the failure is mainly in bond of mortar to gravel. *B* is a section of 1:4.5 concrete, also made with Potomac gravel, tested at 28 days. The average tensile strength was 480 lb/in², and considerable gravel fracture may be seen.

In figure 7, *A* is a section of 1:6.4 concrete made with New York limestone and tested at 28 days. The average tensile strength was 315 lb/in², and very little of the stone was fractured. *B* is a section of 1:4.75 concrete made with the same aggregate and tested at 28 days. Here the tensile strength was 525 lb/in², and some aggregate fracture occurred.

VII. SUMMARY

1. An improved tensile-test method for concrete has been developed. Cylindrical specimens, 4 by 16 in. are used, and loads are applied through threaded rods embedded in a rich mortar at the ends of the specimens. The method gave fairly uniform results because of the greater uniformity of stress distribution over the section of failure than is obtained with grips, or in tapered specimens, etc.

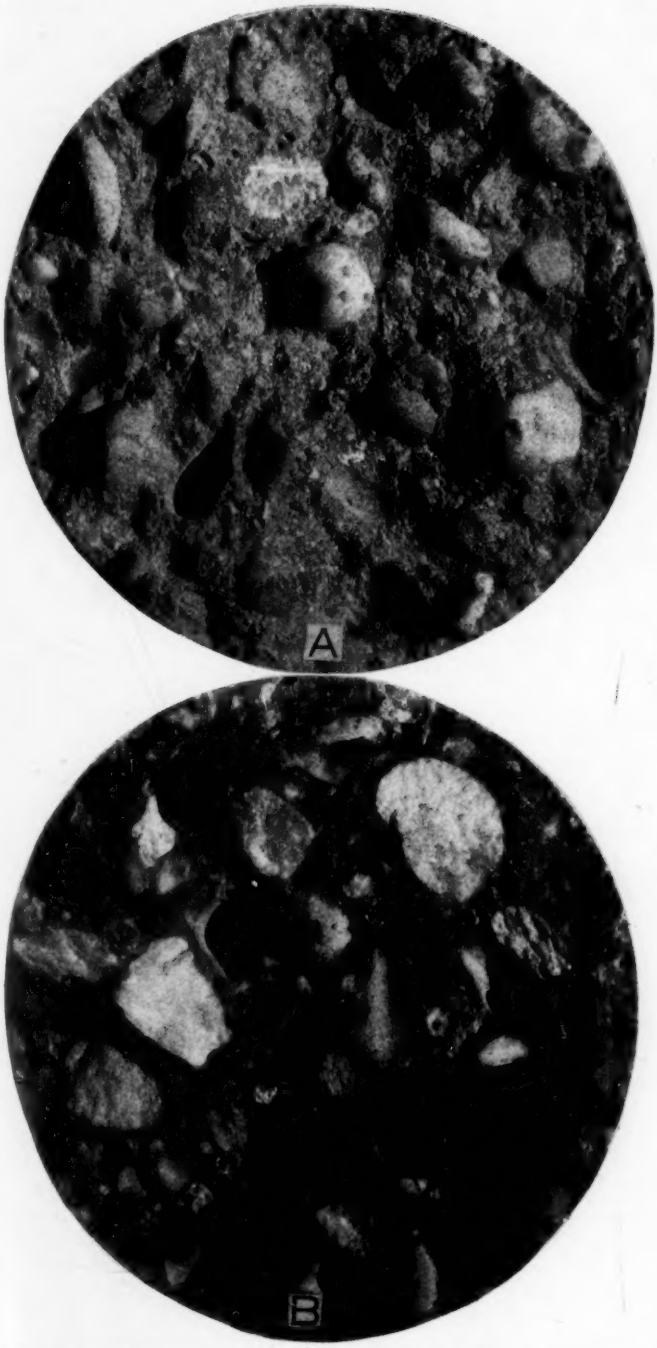

FIGURE 6.—*Fractured sections of Potomac River gravel concrete.*

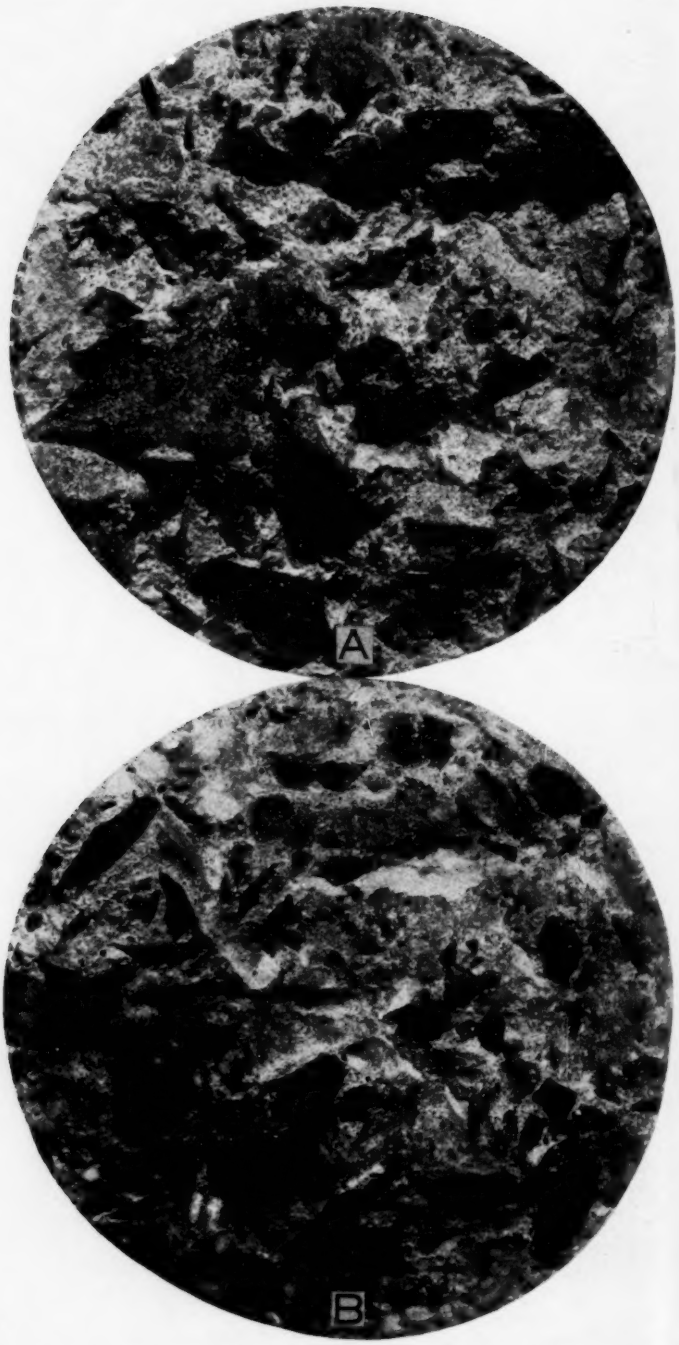

FIGURE 7.—*Fractured sections of New York limestone concrete.*

2. Compressive and tensile strengths and some stress-strain relations were determined for concretes made with a number of cements of various types, including five normal, four moderate-heat, and three high-early-strength portland cements, two portland-pozzolans, one high-alumina cement, and nine portland cements with aerating agents. The effects of various sand-gravel ratios and of various coarse aggregates on tensile strength were also studied.

3. The high-early-strength cements gave higher strengths than the normal cements, both in tension and compression, up to 28 days. The moderate-heat cements gave but slightly lower strengths than the normal cements up to 28 days; thereafter there was some tendency for the moderate-heat cements to give higher compressive and tensile strengths than the normal cements. The cements with aerating agents gave somewhat higher tensile strengths than the normal cements up to 7 days, but thereafter both compressive and tensile strengths were lower than those for the normal cements. Of the two portland-pozzolans tested, one gave rather low early strength but came within the range for the normal cements at later ages; the other gave early strengths comparable to a moderate-heat portland but gave relatively low strengths at later ages. The high-alumina cement gave very high strengths at 1 day but showed comparatively little gain at later ages.

4. Tensile strengths usually attained maximum values between 7 days and 3 months, with retrogression in many cases. However, at 1 year, tensile strengths were generally within ± 10 percent of the 28-day strengths, except for the high-alumina cement, which gave appreciably lower values. The fracture sections of the tensile specimens often showed considerable aggregate breakage at 28 days, and somewhat less at later ages.

5. The 1:4.5 mix generally gave higher strengths for all cements than the 1:6 mix at the same age. The use of either a richer mix with the normal cements, or of a high-early-strength cement, was of greater advantage in securing high early tensile strength than high early compressive strength.

6. Tensile and compressive strengths show a fair relation, but the 1:4.5 mix gave somewhat higher tensile strengths for equal compressive strengths than the 1:6 mix. For compressive strengths up to about 2,500 lb/in.^{2}, the 1:6 aerated concretes gave somewhat higher tensile strengths for equal compressive strengths than the 1:6 mix with normal cements. The high-alumina cement gave appreciably lower tensile strengths for equal compressive strengths than the other cements.

7. For the nine aerated 1:6 concretes tested with Potomac River sand and gravel, the weights per cubic foot ranged from 138 to 144 lb. compared with 146 for a corresponding normal cement concrete. Initial tangent moduli of elasticity for the aerated concretes ranged from 3.4 to 3.9×10^6 lb/in.^{2} in compression, and from 3.3 to 3.8×10^6 lb/in.^{2} in tension, compared with 4.1 and 4.0, respectively, for a normal cement concrete. Both the tensile and compressive moduli increased with compressive strengths, which ranged from 3,100 to 4,600 lb/in.^{2} at 28 days.

8. Tensile strengths of concrete varied considerably with the sand-gravel ratio or with the type of coarse aggregate used. A range of 1.4 to 1 in tensile strength at 28 days was noted for different coarse

aggregates, such as slag, crushed stones, and gravel. A crushed limestone gave relatively low tensile strength in a 1:6.4 mix, and rather high strength in a 1:4.8 mix.

Acknowledgments are due the various cement manufacturers, the National Crushed Stone Association, and the National Slag Association for the materials contributed for use in this investigation.

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WASHINGTON, March 24, 1943.

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2.500 to 5.000	.001	15
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[Continued on p. 4 of cover]

[Continued from p. 3 of cover]

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Range of x	Interval between successive arguments
0 to 7	0.001
7 to 50	.01
50 to 300	.1
300 to 2,000	1
2,000 to 10,000	10

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